# GCE Advanced Level 2014

Combined Mathematics II

Model Paper 05 Time 3 hrs

**PART A**

**(**Answer all questions**)**

1. Using mathematical induction show that $\sin\theta + \sin 2\theta + \cdots + \sin n\theta = \frac{\sin\frac{n\theta}{2}\sin\frac{(n+1)\theta}{2}}{\sin\frac{\theta}{2}}$ $n \in z^+$
2. The term independent from x in the expansion of$\left(\sqrt{x} - \frac{k}{x^2}\right)^{10}$ is 405.find out k.
3. Let $Z = 1 + i$.Find out $a, b$ real constants if $\frac{a}{b+z} = -3 + i$.
4. Find $x \xrightarrow{lim} 1 \left[\frac{x^a - x^b}{x^c - x^d}\right]$.
5. $A = \begin{pmatrix} -2 & 3 & -3 \\ 0 & 1 & 0 \\ 1 & -1 & 2 \end{pmatrix}$ show that $A^{-1} = A$.
6. Using integration by parts find out$\int x\left(\frac{x}{2} - 1\right)^{10} dx$.
7. Given that $A \equiv (4, -1) and\ B \equiv (0,5)$, find out point C which divides AB externally in the ratio 1:3.Get equation of the line perpendicular to AB through C.
8. Write down equations of circles touch the Y-axis at (0, 4) and having X intercept of unit 6.
9. Find out locus of Q, if the chord of contact to the $S = x^2 + y^2 - 4x - 6y - 3 = 0$ from Q always touches the $x^2 + y^2 - 4x - 6y + 4 = 0$.
10. Prove that $\cos 2\theta + \cos 4\theta + \cos 6\theta + 1 = 4\cos\theta\cos 2\theta\cos 3\theta$.Hence solve $\cos 2\theta + \cos 4\theta + \cos 6\theta + 1 = 0$.

**PART B**

**(**Answer only **5** questions**)**

11.

a. Let $a \in R$. Find out range of $a$ for which roots of $(x+a+2)^2+(x-a)^2=2$ are real. If α, β are the roots of above equation, find the equation of roots $3\alpha+2$ $and$ $3\beta+2$. Find out $a$ for which $|\alpha-\beta|=2$

b. Given that k is non zero integer and (x-k) is factor of $f(x)=2x^3-x^2-2x+k$; find out k. For this value of k

   i. Write f(x) as a product of linear factors

   ii. Find out a, b, c in $f(x)\equiv(x-a)(2x+1)(x-1)+bx+c$.

12.

a. Show that $\frac{1}{(r+1)^2}<\frac{1}{r(r+2)}$ for $r \in z^+$.

   Using partial fraction or otherwise find out $\sum_{i=0}^{n}\frac{1}{(r+1)(r+2)}$.

   Deduce that $\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\cdots+\frac{1}{(n+1)^2}<\frac{1}{4}$

b. Find out the greatest coefficient in the expansion $(1+x)^{30}$ for x>0.ifthe term with greatest coefficient is greatest term show that $\frac{15}{16}<x<\frac{16}{15}$

13.

a. Find out magnitude and argument of the $\frac{(1+i)^4}{(-1+i)^2}$

b. Express square roots of 12+5i n the form $a+bi$.Given that P and Q represent these square roots show them on an argand diagram. Q/ represent conjugate of Q. Find out complex number represent by R given that OPR Q/ is rhombus.

c. Z is on argand diagram such that$|z-4-4i|\leq 2$ , $Re(z)\geq 4, Im(z)\geq 5$ $and$ $\arg(z)\geq\frac{\pi}{4}$. Identify the region of Z and show that its area is $\frac{\pi-1}{2}$.

14.

a. For the rectangle shown in figure CD= $EF=2x, CF=DE=y$ $and$ $O\hat{A}B=O\hat{B}A=\theta$.

   i. Given that area of OAB Δ is A, show that $A=\left(x+\frac{y}{t}\right)(y+tx)$ where $t=\tan\theta$. if x and y fixed while t varies show that minimum of A is 4xy.

   ii. Given that $OA=OB=l$ show that $y=l\sin\theta-x\tan\theta$.If area of CDEF is S show that $S=2[lx\sin\theta-x^2\tan\theta]$.Given that $l$ and $\theta$ fixed while x varies show that maximum of S is $\frac{1}{2}l^2\sin\theta\cos\theta$

b. Write down equation o f tangent at P $(\alpha,\beta)$ $to$ $y=4x^3-2x^5$.Show that three tangent can be drawn from origin to this curve and obtain the coordinates of points of contacts.

15.

a. Without substitutions evaluate $\int_0^{\pi/2}\frac{1}{1+\cos x}dx$.

Deduce that $\int_0^{\pi/2} \frac{1}{1+\sin x}\,dx$

Using above results show that $\int_0^{\pi/2} \frac{8+5\sin x+3\cos x}{(1+\sin x)(1+\cos x)}\,dx = 8$

b. Evaluate $\int_0^{x} \frac{x\tan x}{\cos x+\sec x}\,dx$

c. $C_1$ and $C_2$ are curves given by $y = x^2$ and $y = \frac{1}{2(4x^2+1)}$.sketch $C_1$ and $C_2$ on same diagram showing intersection points, stationary points and asymptotes. Find out area S bounded by $C_1$ and $C_2$. Find out the volume of the solid formed by rotating S angle of $\pi$ around y axis.

16.

a. Sides of PQR Δ PQ,QR and RP are given by $x - 3y + 5 = 0, x - y + 3 = 0\ and\ 3x - y - 1 = 0$.Line through P perpendicular to QR meets the line through Q parallel to PR at S. find out equation of QS and PS. Hence show that PQRS is a rhombus and find its area.

b. The center of circle S is on 2x-2y+9=0. S and $x^2 + y^2 - 4 = 0$ intersect orthogonally. Show that S goes through two fixed points. Find out coordinates of them. Among the circles through these two points find out,

   i. The circle which touches the X axis
   ii. The circle with least radius

17.

a. Prove that $-7 \le 5\cos\theta + 3\cos\left(\theta - \frac{\pi}{3}\right) \le 7$for $\forall\ \theta \in R$.

Let $f(\theta) = \frac{5}{5\cos\theta+3\cos\left(\theta-\frac{\pi}{3}\right)+8} + 3\cos\left(\theta - \frac{\pi}{3}\right)$ .show that $\frac{1}{3} \le f(\theta) \le 5$ for $\forall\ \theta \in R$.

Solve $f(\theta) = \frac{5}{8}$ for $0 \le \theta \le 2\pi$

b. In the usual notation show that $a\cos B + b\cos B = c$ for any triangle ABC

c. Show that $a^3\cos(B-C) + b^3\cos(C-A) + c^3\cos(A-B) = 3abc$

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# *A/L combined maths 2015 /2016*

*English/Sinhala medium*

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