GCE Advanced Level 2014

Combined Mathematics I

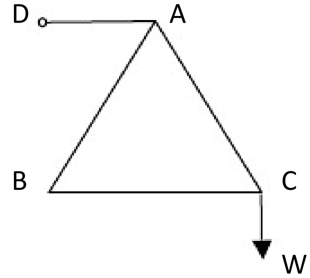
Model Paper 04 Time 3 hrs

PART A

(Answer all questions**)**

1. A cyclist rides along a straight path with uniform velocity u and passes a motor car, which is at rest. At the same instant, the motor car starts to move in the same direction with uniform acceleration a until it attains its greatest velocity v (> u). Draw, in the same diagram, the velocity-time graphs for the motions of the cyclist and the motor car. Deduce that within the period in which the motor car is behind the cyclist, the maximum distance between them is.$\frac{u^2}{2a}$
2. A wedge of mass 2m is free to move on a rough horizontal plane with coefficient of friction μ. A particle of mass m, slides down along a line of greatest slope of the smooth face of the wedge, inclined at an angle α to the horizontal. Find the acceleration of the wedge.
3. A train of mass M is ascending on a smooth track having an inclination of 1 in n. When the velocity of the train is v its acceleration is f. Assuming that the resistance against the motion of the train is negligible, prove that the effective power of the engine is $\frac{Mv}{n}(g + nf)$
4. A smooth small ball of mass m, which is at rest, falls under gravity from a height $8h$ to a horizontal smooth plane and rebounds to a height $2h$. Find the coefficient of restitution between the ball and the plane.Show that the loss of kinetic energy due to the impact is $6mgh$ and find the impulse between the ball and the plane.

5. A particle of mass m is attached to one end of an elastic string of natural length a and modulus of elasticity λ . The other end of the string is attached to a fixed point O on a smooth horizontal plane. Initially the particle is kept at rest at a distance $2a$ from the point O and then released. Show that it is in a simple harmonic motion for a time $\frac{\pi}{2}\sqrt{\frac{ma}{\lambda}}$
6. Forces of magnitude 2P, 3P and 4P act respectively along the sides AB, BC and CA of an equilateral triangle ABC of side. Find the magnitude and the direction of the resultant. Find also, the distance from A to the point where the line of action of the resultant meets AC.
7. The figure shows a framework which consists of four smoothly jointed light rods carrying a weight W at the point C. AB, BC and AC are of equal length. The framework is smoothly hinged at a fixed point D with B resting against a smooth support such that the rods AD and BC are horizontal while B lies vertically below D. The framework is kept in equilibrium in a vertical plane. Using Bow's notation, find graphically, the forces acting on the rods AC, BC and AB.



Tel. No. 0775811667

8. A uniform right circular cylinder of height 40 cm and base radius 6 cm, is placed on a rough horizontal plane with its axis perpendicular to the plane. The coefficient of friction between the cylinder and the plane is 0.4. Determine whether the cylinder slides first or topples first, if the horizontal plane is slowly tilted.
9. The events A and B are such that P(A) = 1/3, P(B) =2/5 and P(A|B ′) =11/20, where B ′ is the complementary event of B.

   Find (i) P(A ∩ B),

   (ii) P(A ∪ B),

   (iii) P(A′|B).
10. The mean and the standard deviation of the marks obtained in an examination by a group of students are 42 and 15 respectively. The marks are now adjusted by using a linear scale so that the mean and standard deviation become 50 and 20 respectively. Find the adjusted mark of a student who scores 54 marks at the examination.

PART B

(Answer only **5** questions**)**

11.

a) An express train travels from station A to its next stop at station B. The distance between the two stations is d km. The uniform acceleration and retardation of the train are f kms^{-2} and λf kms^{-2} respectively, where λ is a positive constant. The greatest velocity that the train can attain and maintain is v kms^{-1}. The train starts from station A at rest and stops at station B in a minimum time.

i. Draw the velocity-time graph for the motion of the train. Hence, show that, if$d \geq \frac{v^2}{2f}\left(1+\frac{1}{\lambda}\right)$ then, the total time taken for the train to reach station B is $\frac{d}{v}+\frac{v}{2f}\left(1+\right.$ 1λ.

ii. If $d < \frac{v^2}{2f}\left(1+\frac{1}{\lambda}\right)$, make the necessary modification in your velocity-time graph and hence, show that in this case, the total time for the train to reach station B is $\sqrt{\frac{2d}{f}\left(1+\frac{1}{\lambda}\right)}$. Find the average speed of the train.

b) A person travelling due East with velocity u, feels the wind blowing from an acute angle α North of East. When he starts travelling due North with velocity 2u, the wind appears to blow from an acute angle β North of East. Draw, in the same figure, the velocity triangles for both cases. Hence, find the direction of the wind

12.

a) A particle is projected under gravity from a point O on the ground with speed u at an angle α to the horizontal, in a plane perpendicular to a wall at a distance a from O . If the particle is at a height y when it is at a horizontal distance x from O show that $gx^2 sec\ \alpha = 2u^2\ (x\ sin\ \alpha - y\ cos\ \alpha\ )$. If the particle just passes over the wall and falls on the ground at a point of distance d from the wall, show that the height of the wall is $\frac{ad}{a+d}\tan\alpha$. Find the maximum height attained by the particle in terms of a, d and α .

b) A light inextensible string passes over a smooth pulley fixed to a ceiling and has a particle of mass M attached to an end and a light smooth pulley attached to the other end. Another light inextensible string passes over the second pulley and carries a mass m1 at one end and a mass m2 at the other end. If the system is released from rest, show that the particle of mass M will remain at rest provided that $\frac{1}{M} = \frac{1}{m_1} + \frac{1}{m_2}$

13. One end of a light elastic string is attached to a fixed point of a ceiling and the other end to a particle, which hangs in equilibrium and causes an extension l in the string. The equilibrium of the particle is disturbed, at time t = 0, by giving it a velocity $2gl$ vertically downwards. Prove that

a) the maximum extension of the string is $3l$

b) the string becomes slack after a time $\frac{7\pi}{6}\sqrt{\frac{l}{g}}$

c) the particle will not hit the ceiling, provided that the natural length of the string exceeds $\frac{3l}{2}$

d) the time needed for the particle to reach its maximum height is$\Big(\sqrt{3} +$ $7\pi 6lg$ when the natural length of the string is greater than $3l2$

14. A smooth hollow right circular cylinder of radius a is fixed with its axis horizontal. Let O be a point on the axis of the cylinder and A, a point on the inner surface of the cylinder with OA being horizontal and perpendicular to the axis of the cylinder. A smooth particle of mass m is projected from A vertically downwards with speed $\sqrt{10ag}$. When the particle P reaches the lowest point of the surface it collides directly with a smooth particle Q of mass 2m which is at rest. The coefficient of restitution is ½.

a) Find the velocity of the particle Q just after the collision.

b) Find the reaction between the particle Q and the inner surface of the cylinder when OQ makes an acute angle θ with the upward vertical.

c) Deduce that the particle Q leaves the surface when OQ makes an angle $\cos^{-1}\left(\frac{1}{3}\right)$ with the upward vertical.

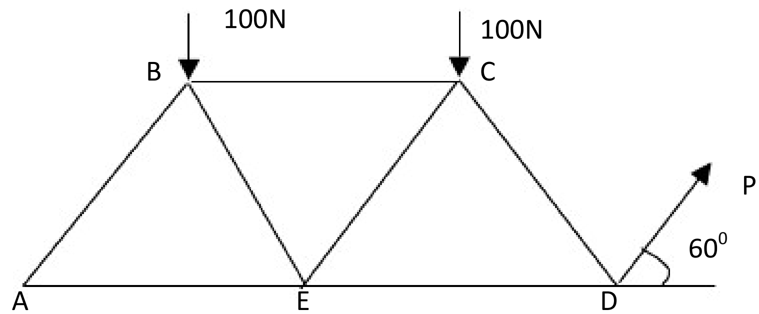
d) Show that the maximum height that the particle Q reaches above the horizontal level through O is $\frac{13a}{27}$

Tel. No. 0775811667

15\.

a) Find the position of the centre of gravity of a solid uniform right circular cone. The base radius and height of a solid uniform right circular cone are a and h respectively. The ends of a light inextensible string of length l which passes over two fixed smooth pegs on the same horizontal level, are connected, one to the vertex and the other to a point on the circumference of the base of the cone. If the cone is in equilibrium with the axis of the cone horizontal, prove that $h^2\ (l - h)(l + h - 2d) = 4a^2\ (h - d)^2$, where d is the distance between the two pegs.

b) The figure below shows a framework consisting of seven smoothly jointed light uniform rods of equal length:



The framework is smoothly hinged at A and two weights of 100 N each are placed at B and C. It is kept in equilibrium in a vertical plane with the rods AE, ED and B C horizontal, by a force P applied at D in the direction which makes an angle 60o to the horizontal. Find the magnitude of P.
Using Bow's notation determine the stresses in the rods B C , C D and ED, distinguishing between tensions and thrusts.

16\. The foot of a uniform ladder of weight 2w and length 4a rests on a rough horizontal ground, and the top of the ladder rests against a smooth vertical wall. The ladder is in equilibrium in a vertical plane, making an angle θ with the downward vertical. The coefficient of friction between the ladder and the ground is μ.

a) Show that a man of weight 6w can climb safely to the top of the ladder provided that $\mu \geq \frac{7}{8} tan\ \theta$.

b) Suppose that $\theta = \frac{\pi}{6}\ and\ \mu < \frac{\sqrt{3}}{8}$

i. Show that the minimum couple required to be applied to the ladder, for the man to climb safely to the top of the ladder is $8aw$.

ii. Find the maximum distance that he can climb along the ladder, carrying a weight of w.

17. Drivers are classified by an insurance company as low, average or high risk drivers. The company estimates that at present they have 25% low, 60% average and 15% high risk drivers in their records. The probabilities of such drivers encountering a given number of accidents during a year are indicated in the following table:

		Risk		
		Low	Average	High
No of accidents per year	0	x	0.93	0.74
	1	0.01	y	0.10
	2	0.00	0.01	z
	3	0.00	0.00	0.01
	>=4	0.00	0.00	0.00

i. Find the appropriate values of x, y and z.
ii. Find the probability that a randomly selected driver had no accident in the year.
iii. If A had no accidents in the year, find the probability that he is a high-risk driver.
iv. If B had no accidents for 4 such years, find the probability that he is a low risk driver.

b) The sum and the sum of squares of the times (in minutes) taken by 20 students to do their homework in Mathematics are 320 and 5840 respectively.

i. Calculate the mean and the standard deviation of the distribution of times taken by the 20 students to do their homework in Mathematics.
ii. The time taken by another student to do his homework in Mathematics is added and it is found that the mean is unchanged. Show that the standard deviation is decreased.
iii. The sum and the sum of squares of the times (in minutes) taken by another 10 students to do their homework in Mathematics are 130 and 2380 respectively.
iv. Find the mean and the standard deviation of the times taken by all 30 students.

A/L Combined Math , Edexcel ,Cambridge AS / A2 classes

by result oriented Bsc. Engineer

Home visited in Colombo

Voice : 0775811667