GCE Advanced Level 2014

Combined Mathematics I

Model Paper 01 Time 3 hrs

PART A

(Answer all question**)**

1. Prove by the method of mathematical induction ,that $2^1 + 2^2 + 2^3 + \cdots + 2^n = 2(2^n - 1)$
2. Given that $(x - 1)$ is a factor of $2x^3 - 9x^2 + 3x + k$, find the value of k and factorize the expression completely. Hence, find out factors of $4x^3 + 3x^2 - 9x + 2$.
3. The coefficient of x^7 and x^8 in the expansion of $\left(3 + \frac{x}{2}\right)^n$ are equal. Find the value of n.
4. Express $z = \frac{5(1-i)^3}{(2+i)(1+2i)}$ in the form $x + iy$.Hence show that Z^4 is purely real.
5. Given that, $x + y = 2a$ and $x, y \in z^+$.Show that sum of squares of x and y is minimum for x=y and $2a^2$ is the minimum.
6. Evaluate $\int x \sin x \cos x \, sec^3 x$ using integration by parts.
7. Straight line $l = 0$ is through origin and meets $x + y = 1$ and $x + y = 3$ at P and Q respectively.Two lines $l_1 and\ l_2$ are through P and Q parallel to $2x - y = 5$ and $3x + y = 5$ respectively. Find out the locus of the intersection point of $l_1 and\ l_2$.
8. The circle $x^2 + y^2 - 4x - 6y + k = 0$ neither intersects nor touches the x and y-axis and (2,2) is inside the circle.. show that $9 < k < 12$
9. Write down the equation of the circle with radius of three units and touches $x^2 + y^2 - 4x - 6y - 12 = 0$ internally at (-1,-1) .
10. Referring to usual notation of a triangle ABC, $a = 2\sqrt{2},\ b = 2\sqrt{3}\ and\ A = 45^0$. Show that two values can be found for B. Deduce that length of AB is $\sqrt{6} + \sqrt{2}\ or\ \sqrt{6} - \sqrt{2}$

PART B

(Answer only **5** questions**)**

11.

a. Given that, $f(x) = (n+1)x + \frac{(n-1)}{x}$ and x is none zero real number and n is integer greater than one. Show that value of f(x) can't be lie in between $-2\sqrt{n^2-1}$ and$2\sqrt{n^2-1}$. if $k > 2\sqrt{n^2-1}$ show that both roots of f(x)=k are positive.

b. Express and prove the remainder theorem for polynomials. P(x) is polynomial of degree three and coefficient of x^3 is 1.When P(x) is divided by (x-1) and (x-3) remainders are 7 and 13 respectively. Find out the remainder when it is divided by (x-1)(x-3).If P(2)=6 find out the quotient when P(x) divided by (x-1)(x-3). Hence obtain P(x).

12.

a. Using expansion of $(r+1)^3$-r^3 or other method show that $\sum_{r=1}^{n} r^2 = \frac{n}{6}(n+1)(2n+1)$

Find out $\sum_{r=1}^{n} \frac{1}{r(r+1)}$

Write down r [th] term U_r of $\frac{3}{1^2} + \frac{5}{1^2+2^2} + \frac{7}{1^2+2^2+3^2} + \frac{9}{1^2+2^2+3^2+4^2} + \cdots$

Find out $\sum_{r=1}^{n} U_r$.

Find out the least positive integer of n for $6 - \sum_{r=1}^{n} U_r < \frac{1}{1000}$

b. $u_1, u_2, u_3 \ldots . u_n$ is an arithmetic series of the positive numbers. By method of mathematical induction for $n \geq 2$ show that $\frac{1}{u_1 u_2} + \frac{1}{u_2 u_3} + \frac{1}{u_3 u_4} + \cdots + \frac{1}{u_{n-1} u_n} = \frac{n-1}{u_1 u_n}$

13.

a. Write down roots of $x^2 + x + 1 = 0$.if one root is ω show that other one is ω^2.

   i. Show that $(a + b\omega + c\omega^2)(a + c\omega + b\omega^2) = a^2 + b^2 + c^2 - (ab + bc + ac)$

   ii. Show that vale of $\sqrt{-1 - \sqrt{-1 - \sqrt{-1 - \sqrt{\ldots .}}}}$ is $\omega \; or \omega^2$

   iii. Given that Z_1,Z_2 and Z_3 are three complex numbers and $A = z_1 + z_2 + z_3; B = z_1 + \omega z_2 + \omega^2 z_3 \; and \; C = z_1 + \omega^2 z_2 + \omega z_3$ .Express Z_1,Z_2 and Z_3 in terms of A,B C and ω.

b. Let Z=1+i

   i. Given that $\frac{a}{b+Z} = -3 + i$, find out real constant a and b

   ii. Obtain the magnitude and argument of Z^3

   iii. A represent Z on argand plane and B represent Z^5.If O is origin find out the complex number given by C as OACB is parallelogram. Find out length of diagonal AB.

14.

a. Evaluate $\lim_{x \to 0} \frac{2^x - \cos x}{\sin x}$

b. Write down equation of the tangent and normal at (1,1) to the curve $y^2 = x(2-x)^2$ Show that tangent meets the curve again and find out meting point

c. Coordinates of point P on $y^2 = 4ax\ (a > 0)$ is given by $(at^2, 2at)$ where t is parameter. Line drawn from P as parallel to y axis intersects curve again at Q. write down coordinates of Q in term of t.(use symmetry).M and N are the foots of perpendiculars drawn respectively from P and Q to the line goes through (a,0) and parallel to PQ . Show that maximum area of the rectangle PQMN is $\frac{8a^2}{3\sqrt{3}}$.

15.

a. Given that $\cos\alpha + \cos\beta = 1\ and\ \sin\alpha + \sin\beta = k$,find out $\cos(\alpha - \beta)$ and $\sin(\alpha + \beta)$.Hence show that $-\sqrt{3} \leq k \leq \sqrt{3}\ for\ all\ \alpha, \beta$

b. Let $f(\theta) = sin^6\theta + cos^6\theta$; $and\ \theta\ is\ real.$ Express $f(\theta)\ in\ the\ form\ A + B\cos k\theta$ where A, B and k are constant. Hence solve $4(sin^6\theta + cos^6\theta) - 2\sin 4\theta - 5 = 0$

c. Express cosine rule for a triangle. In usual notation for a ABC triangle show that $\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c} = \frac{a^2+b^2+c^2}{2abc}$
If $\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c} = k\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$ find out least value of k.

16.

a. Find out λ and μ as $\sin x = \lambda(\sin x + \cos x) + \mu(\cos x - \sin x)$ hence or otherwise evaluate $\int \frac{\sin x}{\sin x + \cos x} dx$ and $\int \frac{\cos x}{\sin x + \cos x} dx$

b. By integration of parts evaluate $\int \frac{\ln x}{(1+x)^3} dx$

c. Find out area of the region satisfies $x^2 + y^2 < 2x, x + y \leq 1\ and\ y \geq 0$

17.

a. ABCD is a rhombus and equation of AB is 4x-3y+15=0.BD diagonal is 2x+y-5=0 and A=(-3,1).Find out equations of AC and other sides of the rhombus.

b. Given that t(α,β) is a point on $x^2 + y^2 + 2gx + 2fy + c = 0$ show that equation of the tangent to the circle at t is $(\alpha + g)x + (\beta + f)y + \alpha g + \beta f + c = 0$.
If it goes through origin deduce $\alpha g + \beta f + c = 0$
Using above results finds out the coordinates of the contact point A and B of the tangent drawn from origin to the circle $x^2 + y^2 - 6x - 2y + 9 = 0$.
Write down equation of AB.
Hence or otherwise show that circle through A,B and origin is $x^2 + y^2 - 3x - y = 0$.

AL combined Math, Cambridge Edexcel AS A2 math

Theory and Paper

Classes by result oriented

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