Combined Mathematics II - Part B

Model Answers

11.

(a) A particle P is projected at a point O vertically upwards under the gravity with velocity u. After time $\frac{u}{2g}$, another particle Q is projected at the point O vertically upwards under the gravity with velocity $v(>u)$. Let A be the highest point that the particle P reaches. The particles P and Q meet at the point A. Draw the velocity-time graphs for the complete motions of the particles P and Q **in the same figure**. Using these velocity-time graphs show that

(i) $OA = \frac{u^2}{2g}$,

(ii) $v = \frac{5u}{4}$ and the velocity of the particle Q at the point A is $\frac{3u}{4}$.

(iii) when the particle Q reaches the highest point the height of the particle P from the point O, is $\frac{7u^2}{32g}$

(b) A car of mass M kg is travelling on a level road against a resistance R of the motion which is a constant at all speeds. If the maximum power of the engine is H kW and the car has a maximum speed of v ms^{-1} on a level road, find the resistance R in terms of M, H and v.
Find the acceleration of the car in terms of M, H, v, g and α when it is moving

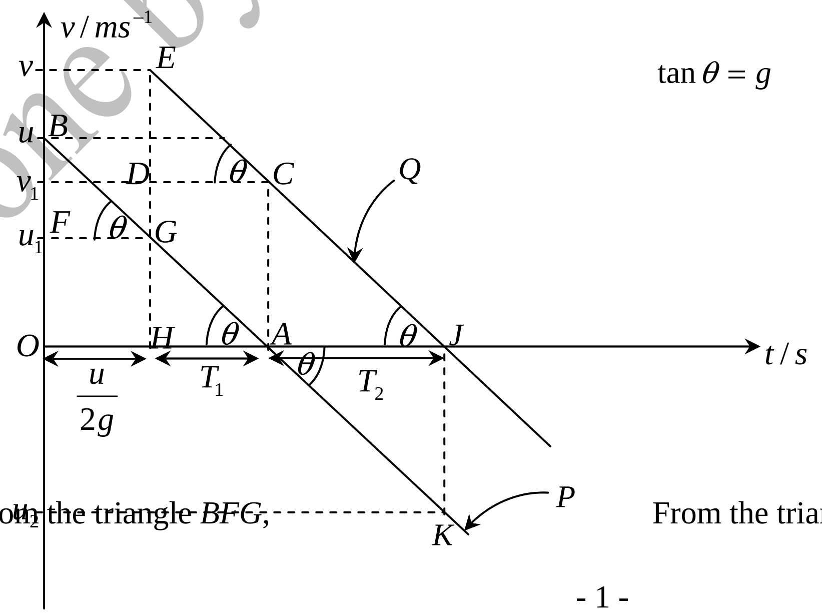
(i) at speed $\frac{v}{3}$ ms^{-1} directly up,

(ii) at speed $\frac{v}{2}$ ms^{-1} directly down

along a straight road inclined at an angle α to the horizontal.
If the acceleration of the car in case (ii) is twice that in case (i), find $\sin\alpha$ in terms of M, H, v and g.
In this case find the maximum speed in terms of v that can be made by the car when moving directly up the road.

Answer

(a)



$\tan\theta = g$

From the triangle BFG,

From the triangle GHA,

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$$\tan\theta = \frac{u-u_1}{\frac{u}{2g}}$$

$$\tan\theta = \frac{u_1}{T_1}$$

$$\Rightarrow g\times\frac{u}{2g} = u-u_1$$

$$\Rightarrow g = \frac{\frac{u}{2}}{T_1}$$

$$\Rightarrow u_1 = \frac{u}{2}$$

$$\Rightarrow T_1 = \frac{u}{2g}$$

$\therefore\ OA =$ Area of the triangle ABO

$$= \frac{1}{2}\times u\times\left(\frac{u}{2g}+T_1\right)$$

$$= \frac{u}{2}\times\left(\frac{u}{2g}+\frac{u}{2g}\right)$$

$$= \frac{u}{2}\times\frac{u}{g}$$

(i) $OA = \dfrac{u^2}{2g}$

From the triangle EDC,

$$\tan\theta = \frac{v-u_1}{T_1}$$

$$\Rightarrow v - v_1 = g\times\frac{u}{2g} = \frac{u}{2} ------(1)$$

When the particles P and Q meet at the point A,
Area of $ECAH$ = Area of the triangle ABO

$$\Rightarrow \frac{1}{2}\times T_1\times(v+v_1) = \frac{1}{2}\times\left(\frac{u}{2g}+\frac{u}{2g}\right)\times u$$

$$\Rightarrow \frac{u}{2g}\times(v+v_1) = \frac{u}{g}\times u$$

$$\Rightarrow v+v_1 = 2u ------(2)$$

$$v - v_1 = \frac{u}{2} ------(1)$$

$(1)+(2);\ 2v = \dfrac{5u}{2} \Rightarrow v = \dfrac{5u}{4}$

$(2)-(1);\ 2v_1 = \dfrac{3u}{2} \Rightarrow v_1 = \dfrac{3u}{4}$

(ii) $v = \dfrac{5u}{4}$

Velocity of the particle Q at the point $A = \dfrac{3u}{4}$

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From the triangle *EHJ*,

$$\tan\theta = \frac{v}{T_1 + T_2}$$

$$\Rightarrow g = \frac{5u}{4\left(\frac{u}{2g} + T_2\right)}$$

$$\Rightarrow g \times \frac{2u}{g} + 4gT_2 = 5u$$

$$\Rightarrow 4gT_2 = 3u$$

$$\Rightarrow T_2 = \frac{3u}{4g}$$

From the triangle *AJK*,

$$\tan\theta = \frac{u_2}{T_2}$$

$$\Rightarrow u_2 = g \times \frac{3u}{4g}$$

$$\Rightarrow u_2 = \frac{3u}{4}$$

$\therefore$ The height of the particle *P* from *O* when *Q* reaches the highest point $= \frac{1}{2} \times u \times \left(T_1 + \frac{u}{2g}\right) - \frac{1}{2} \times u_2 \times T_2$

$$= \frac{1}{2}\left[u \times \frac{u}{g} - \frac{3u}{4} \times \frac{3u}{4g}\right]$$

$$= \frac{1}{2}\left[\frac{u^2}{g} - \frac{9u^2}{16g}\right]$$

$$= \frac{1}{2} \times \frac{7u^2}{16g}$$

$$= \frac{7u^2}{32g}$$

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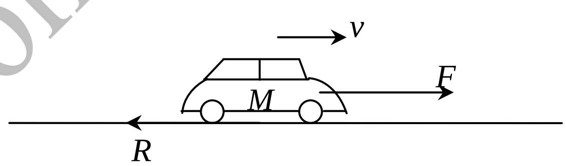
(b)



Let *F* is the tractive force of the car.

Power $=$ Velocity $\times$ Tractive force of the engine

$$\Rightarrow 1000H = v \times F$$

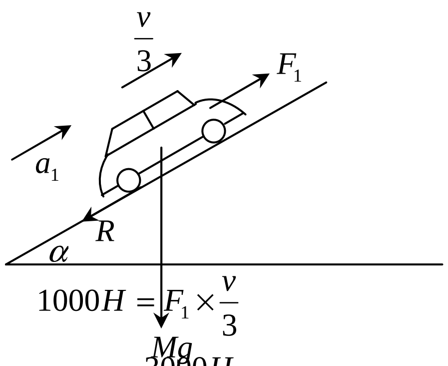
$$\Rightarrow F = \frac{1000H}{v}$$

Applying $\rightarrow \underline{F} = m\underline{a}$ to the motion of the car,

$$F - R = M(0)$$

$$\Rightarrow R = F = \frac{1000H}{v}$$

$$\therefore R = \frac{1000H}{v}$$



Let a_1 - Acceleration of the car

F_1 - Tractive force of the car,

when the car is move at speed $\frac{v}{3}$ ms^{-1} directly up.

$$1000H = F_1 \times \frac{v}{3}$$

$$\Rightarrow F_1 = \frac{3000H}{v}$$

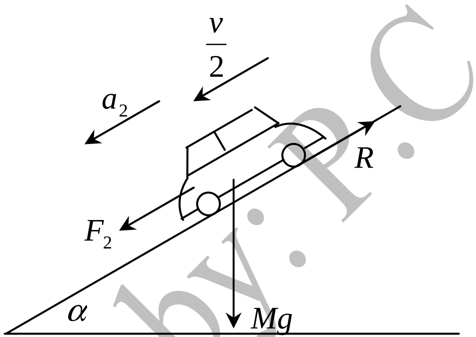
Applying $\nearrow \underline{F} = m\underline{a}$ to the motion of the car,

$$F_1 - R - Mg\sin\alpha = Ma_1$$

$$\Rightarrow \frac{3000H}{v} - \frac{1000H}{v} - Mg\sin\alpha = Ma_1$$

$$\Rightarrow \frac{2000H}{v} - Mg\sin\alpha = Ma_1$$

(i) $\therefore a_1 = \frac{2000H}{Mv} - g\sin\alpha$



Let a_2 - Acceleration of the car

F_2 - Tractive force of the car,

when the car is move at speed $\frac{v}{2}$ ms^{-1} directly down.

$$1000H = F_2 \times \frac{v}{2}$$

$$\Rightarrow F_2 = \frac{2000H}{v}$$

Applying $\swarrow \underline{F} = m\underline{a}$ to the motion of the car,

$$F_2 + Mg\sin\alpha - R = Ma_2$$

$$\Rightarrow \frac{2000H}{v} + Mg\sin\alpha - \frac{1000H}{v} = Ma_2$$

$$\Rightarrow \frac{1000H}{v} + Mg\sin\alpha = Ma_2$$

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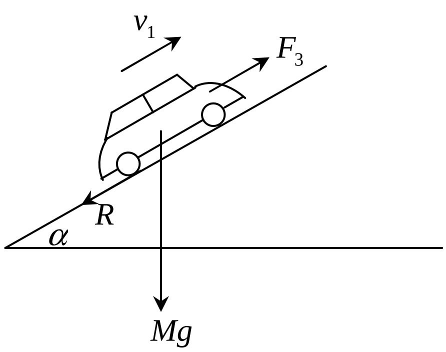
(ii) $\therefore a_2 = \dfrac{1000H}{Mv} + g\sin\alpha$

If $a_2 = 2a_1$,

$$\frac{1000H}{Mv} + g\sin\alpha = \frac{4000H}{Mv} - 2g\sin\alpha$$

$$\Rightarrow 3g\sin\alpha = \frac{3000H}{Mv}$$

$$\therefore \sin\alpha = \frac{1000H}{Mgv}$$



Let v_1 be the maximum speed of the car, when it move directly up the road and F_3 is the tractive force of the car.

Applying $\nearrow \underline{F} = m\underline{a}$ to the motion of the car,

$$F_3 - R - Mg\sin\alpha = M(0)$$

$$\therefore F_3 = R + Mg\sin\alpha$$

$$= \frac{1000H}{v} + Mg \times \frac{1000H}{Mgv}$$

$$F_3 = \frac{2000H}{v}$$

Power $= F_3 v_1$

$$\Rightarrow 1000H = \frac{2000H}{v} \times v_1$$

$$\therefore v_1 = \frac{v}{2}$$

12.

(a) A particle is projected under the gravity in a vertical plane with a velocity *u* at an angle θ to the horizontal, at a point *C* which is at a height *k* from *O*. Consider a rectangular Cartesian system of coordinates by taking horizontal and vertical lines through the point *O* in the plane of projection as *Ox* and *Oy* axes respectively. If at time *t* the particle is at the point (x, y), show that

$$y = k + x\tan\theta - \frac{gx^2\sec^2\theta}{2u^2}.$$

A particle *P* is projected under the gravity in the vertical plane at the point $A(0, h)$, where *h* is positive, with a velocity *v* at an angle α to the horizontal. At the same instant another particle *Q* is projected under the gravity in the same vertical plane at the point $B\left(0, \dfrac{h}{2}\right)$ with a velocity *w* at an angle $\beta(>\alpha)$ to the horizontal. If the two particles *P* and *Q* meet at a point whose horizontal distance is *d*, show that $v\cos\alpha = w\cos\beta$ and $h = 2d(\tan\beta - \tan\alpha)$.

Show also, that the time taken for the two particles to meet is $\dfrac{h}{2(w\sin\beta - v\sin\alpha)}$.

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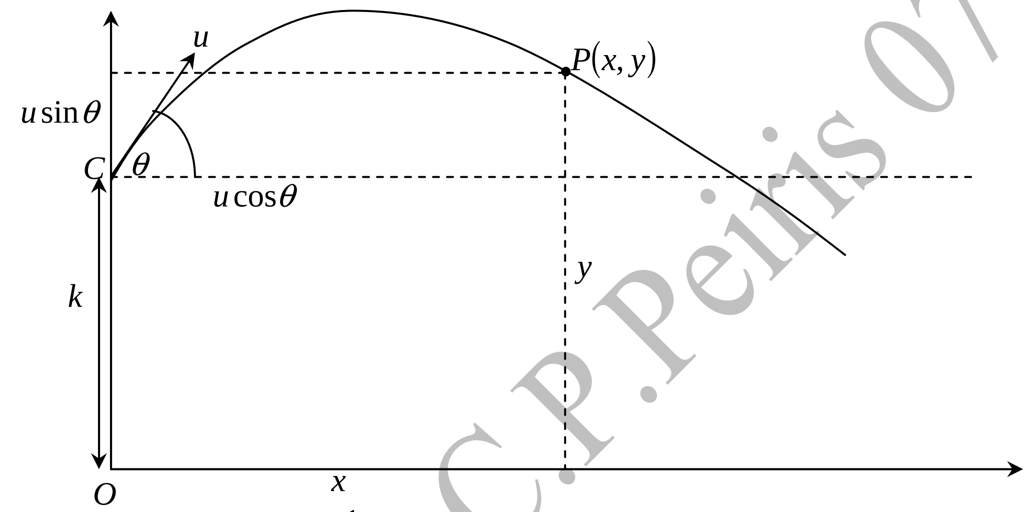
(b) One end of a light inextensible string is attached to a ceiling which is at a height of 3 metres from a horizontal floor. The string passes under a smooth light movable pulley P to which a particle of mass m is fixed and then over a smooth light pulley fixed to the ceiling. A particle Q of mass $M\ (> m)$ is attached to the other end of the string. When the movable pulley P and the particle Q are at heights $\frac{1}{2}$ metres and 1 metres respectively from the floor and the portions of the string not in contact with pulleys are vertical, the system is released from rest.
Find the acceleration of the particle Q and the tension in the string.
Show that the particle Q will reach the floor after time $\sqrt{\frac{4M+m}{(2M-m)g}}$ seconds and the pulley P will rise to height $\frac{1}{2}+\frac{3M}{4M+m}$ metres from the floor.

Answer

(a)



Applying $\rightarrow s = ut + \frac{1}{2}at^2 \qquad C \rightarrow P$

$x = u\cos\theta\, t$

$\Rightarrow t = \dfrac{x}{u\cos\theta}$

Applying $\uparrow s = ut + \frac{1}{2}at^2 \qquad C \rightarrow P$

$y - k = u\sin\theta\, t - \frac{g}{2}t^2$

$\Rightarrow y = k + \dfrac{u\sin\theta}{u\cos\theta}x - \dfrac{g}{2}\dfrac{x^2}{u^2\cos^2\theta}$

$\therefore\ \underline{\underline{y = k + x\tan\theta - \dfrac{gx^2\sec^2\theta}{2u^2}}} \ ------(1)$

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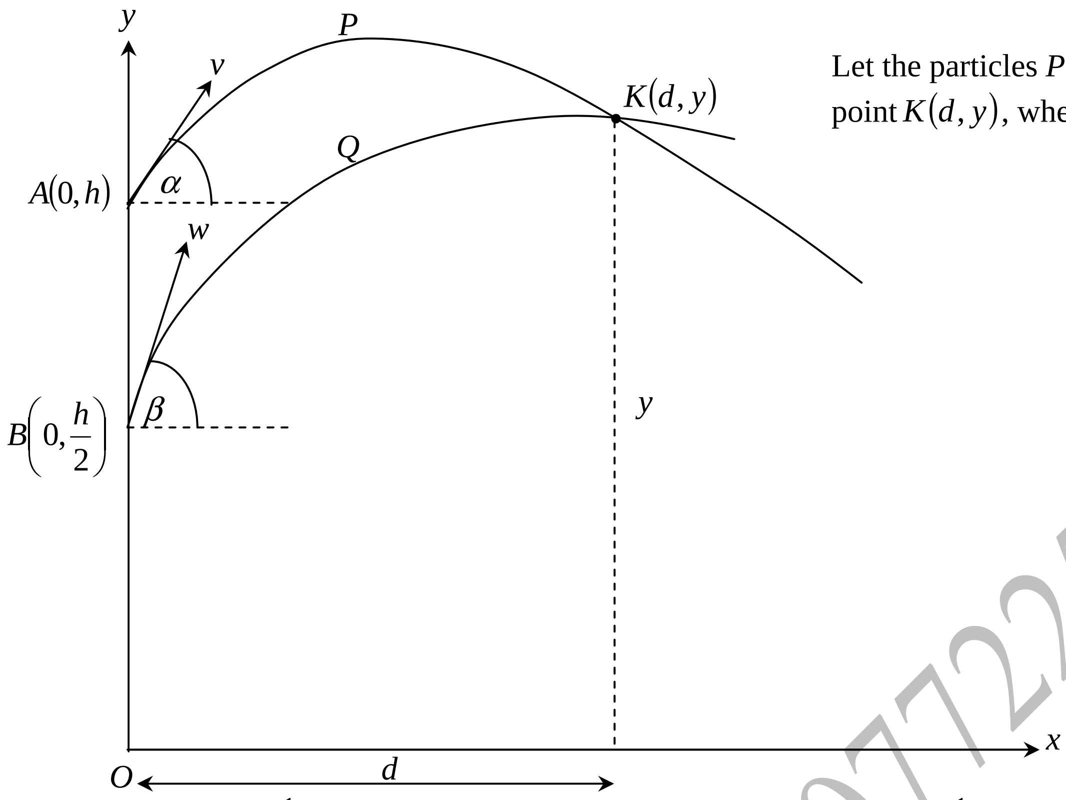

Let the particles P and Q meet at the point $K(d, y)$, when time $= t$

Applying $\rightarrow s = ut + \frac{1}{2}at^2 \quad A \rightarrow \text{K}$ to the particle P

$d = v\cos\alpha\, t - - - - (2)$

Applying $\rightarrow s = ut + \frac{1}{2}at^2 \quad B \rightarrow \text{K}$ to the particle Q

$d = w\cos\beta\, t - - - - (3)$

From (2) and (3); $v\cos\alpha\, t = w\cos\beta\, t$

$$\Rightarrow \underline{\underline{v\cos\alpha = w\cos\beta}}$$

When $u = v$, $\theta = \alpha$, $k = h$ and $x = d$, from the equation (1);

$$y = h + d\tan\alpha - \frac{gd^2\sec^2\alpha}{2v^2} - - - - - (4)$$

When $u = w$, $\theta = \beta$, $k = \frac{h}{2}$ and $x = d$, from the equation (1);

$$y = \frac{h}{2} + d\tan\beta - \frac{gd^2\sec^2\beta}{2w^2} - - - - - - (5)$$

$$(4) - (5);\ 0 = \frac{h}{2} - d(\tan\beta - \tan\alpha) + \frac{gd^2}{2}\left(\frac{1}{w^2\sec^2\beta} - \frac{1}{v^2\sec^2\alpha}\right)$$

$$\Rightarrow 0 = \frac{h}{2} - d(\tan\beta - \tan\alpha) + \frac{gd^2}{2}\left(\frac{v^2\sec^2\alpha - w^2\sec^2\beta}{v^2w^2\sec^2\alpha\sec^2\beta}\right)$$

Since $v\sec\alpha = w\sec\beta$, $v^2\sec^2\alpha - w^2\sec^2\beta = 0$.

$$\therefore 0 = \frac{h}{2} - d(\tan\beta - \tan\alpha)$$

$$\Rightarrow \underline{\underline{h = 2d(\tan\beta - \tan\alpha)}}$$

From (2); $t = \frac{d}{v\cos\alpha} \Rightarrow t = \frac{h}{2v\cos\alpha(\tan\beta - \tan\alpha)}$

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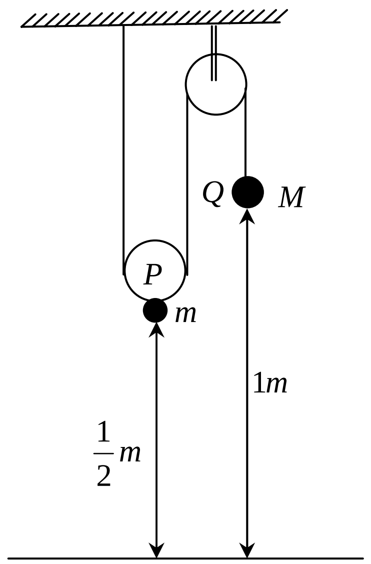
$$\Rightarrow t = \frac{h}{2v\cos\alpha\left(\frac{\sin\beta}{\cos\beta} - \frac{\sin\alpha}{\cos\alpha}\right)}$$

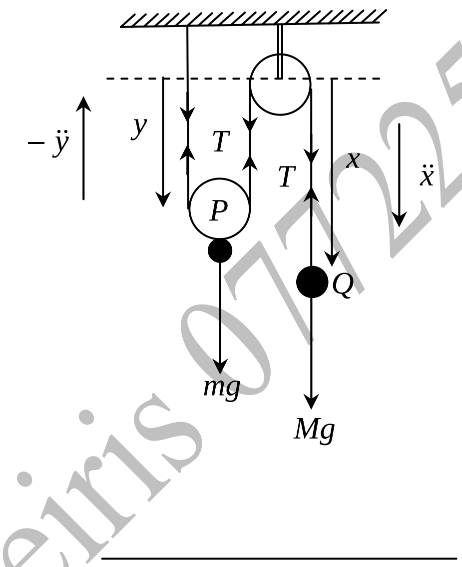
$$\Rightarrow t = \frac{h}{2\left(w\cos\beta\frac{\sin\beta}{\cos\beta} - v\cos\alpha\frac{\sin\alpha}{\cos\alpha}\right)} \quad (\because v\cos\alpha = w\cos\beta)$$

$$\therefore t = \frac{h}{2(w\sin\beta - v\sin\alpha)}$$

(b)



Initial position



General position

Let x be the distance to the particle Q from the fixed line and y be the distance to the movable pulley P from that line.

Since the string is inextensible, $x + 2y = l$, where l is a constant.

$\ddot{x} + 2\ddot{y} = 0 \Rightarrow \ddot{x} = -2\ddot{y}$ ------ (1)

Applying $\downarrow \underline{F} = m\underline{a}$ to the motion of the particle Q,

$Mg - T = M\ddot{x}$ ------ (2)

Applying $\uparrow \underline{F} = m\underline{a}$ to the motion of the movable pulley P,

$2T - mg = -m\ddot{y}$ ------ (3)

$(3) + 2\times(2)$; $2Mg - mg = 2M\ddot{x} - m\ddot{y}$

$$\Rightarrow 2Mg - mg = 2M\ddot{x} - m\left(-\frac{\ddot{x}}{2}\right)$$

$$\Rightarrow 4Mg - 2mg = (4M + m)\ddot{x}$$

$$\Rightarrow \ddot{x} = \frac{2(2M - m)g}{(4M + m)}$$

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From (2); $T = Mg - \dfrac{2M(2M-m)g}{(4M+m)}$ From (1); $\ddot{y} = -\dfrac{(2M-m)g}{(4M+m)}$

$$\Rightarrow T = g\left[\frac{4M^2 + Mm - 4M^2 + 2Mm}{(4M+m)}\right]$$

$$\Rightarrow T = \frac{3Mmg}{(4M+m)}$$

$\therefore$ Acceleration of the particle $Q = \dfrac{2(2M-m)g}{(4M+m)}$

Tension of the string $= \dfrac{3Mmg}{(4M+m)}$

Let t be the time taken to the particle Q to reach horizontal floor.

Applying $\downarrow s = ut + \frac{1}{2}at^2$ to the particle Q

$$1 = 0 + \frac{1}{2}\ddot{x}t^2$$

$$\Rightarrow t^2 = \frac{2}{\ddot{x}}$$

$$\Rightarrow t^2 = \frac{2(4M+m)}{2(2M-m)g}$$

$$\Rightarrow t^2 = \frac{(4M+m)}{(2M-m)g}$$

$$\Rightarrow t = \sqrt{\frac{(4M+m)}{(2M-m)g}}$$

$\therefore$ The particle Q reaches the floor after time $\sqrt{\dfrac{(4M+m)}{(2M-m)g}}$ seconds.

Let h and v be the height from the **initial position** and the velocity of the movable pulley P when the particle Q reaches the floor.

Applying $\uparrow s = ut + \frac{1}{2}at^2$ to the movable pulley P

$$h = 0 - \frac{\ddot{y}}{2}\frac{(4M+m)}{(2M-m)g}$$

$$\Rightarrow h = \frac{1}{2}\frac{(2M-m)g}{(4M+m)}\frac{(4M+m)}{(2M-m)g} = \frac{1}{2}$$

$$\therefore h = \frac{1}{2}\text{ m}$$

Applying $\uparrow v^2 = u^2 + 2as$ to the movable pulley P

$$v^2 = 0 - 2\times\ddot{y}\times\frac{1}{2}$$

$$\Rightarrow v^2 = \frac{(2M-m)}{(4M+m)}g$$

When the particle Q reaches the floor, the string becomes slack then the movable pulley P moves under gravity until it attains its maximum height h_1 from the point where it above $\left(\frac{1}{2} + h\right)$ metres from the floor.

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We have $v=0$, $u^2=\frac{(2M-m)}{(4M+m)}g$, $a=g$ and $s=h_1$

Applying $\uparrow v^2=u^2+2as$ to the movable pulley P

$$0=\frac{(2M-m)}{(4M+m)}g-2gh_1$$

$$\Rightarrow h_1=\frac{1}{2}\frac{(2M-m)}{(4M+m)}$$

$\therefore$ Height to the movable pulley P from the floor $=\frac{1}{2}+h+h_1$

$$=\frac{1}{2}+\frac{1}{2}+\frac{1}{2}\frac{(2M-m)}{(4M+m)}$$

$$=\frac{1}{2}+\frac{1}{2}\left(\frac{4M+m+2M-m}{4M+m}\right)$$

$$=\frac{1}{2}+\frac{1}{2}\left(\frac{6M}{4M+m}\right)$$

$$=\frac{1}{2}+\frac{3M}{4M+m}$$

13. A and B are two points on a smooth horizontal table at a distance $8l$ apart. A smooth particle P of mass m lies at a point on AB in between the points A and B. The particle P is attached to the point A by a light elastic string of natural length $3l$ and modulus of elasticity 4λ and to the point B by a light elastic string of natural length $2l$ and modulus of elasticity λ.

If the particle P is in equilibrium at a point C, show that $AC=\frac{42}{11}l$.

The particle P is held at the mid-point M of AB and then is released from rest. When the particle P is at a distance x from the point A along AB, obtain the tension of the two strings.

Write down the equation of motion of the particle P for $\frac{40}{11}l\le x\le 4l$ and show that, in the usual notation, that $\ddot{x}+\frac{11\lambda}{6ml}\left(x-\frac{42}{11}l\right)=0$.

By writing $y=x-\frac{42}{11}l$, show that $\ddot{y}+\frac{11\lambda}{6ml}y=0$.

Assuming that the solution of the above equation is of the form $y=A\cos\omega t+B\sin\omega t$, find the constants A, B and ω.

Find the velocity of the particle P when it is at a point, distance $\frac{41}{11}l$ from the point.

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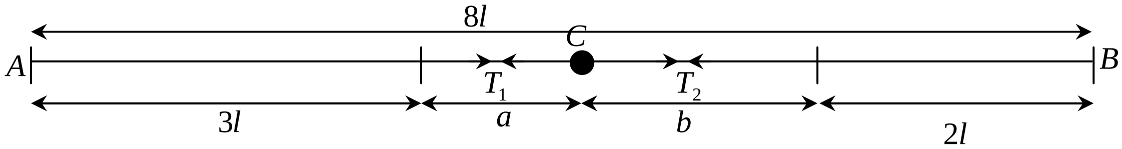
Answer



Equilibrium position

Let a and b are the extensions of the strings AC and BC respectively.
Let T_1 and T_2 are the tensions of the strings AC and BC respectively.
Since the length $AB = 8l$, $3l + a + b + 2l = 8l \Rightarrow a + b = 3l$ ------(1)

From the Hooke's law,

$$T_1 = \frac{4\lambda a}{3l} \qquad\qquad T_2 = \frac{\lambda b}{2l}$$

When the particle is in the equilibrium at the point C,

$T_1 = T_2 \Rightarrow \dfrac{4\lambda a}{3l} = \dfrac{\lambda b}{2l}$

$\Rightarrow \dfrac{4a}{3} = \dfrac{b}{2}$

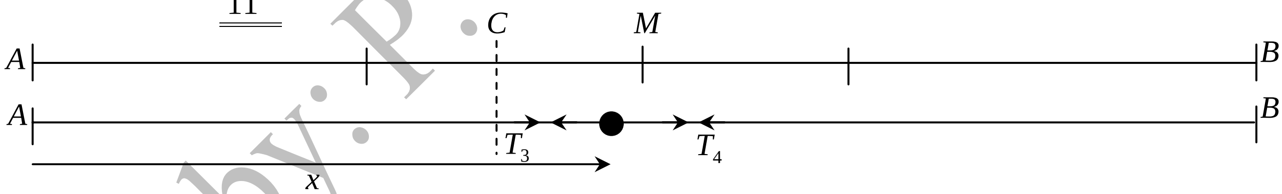
$\Rightarrow 8a - 3b = 0$ -------(2)

$(1)\times 3 + (2);\ 11a = 9l$

$$\Rightarrow a = \frac{9l}{11} \qquad\qquad \therefore b = \frac{24l}{11}$$

$\therefore AC = 3l + a = 3l + \dfrac{9l}{11}$

$\Rightarrow AC = \dfrac{42}{11}l$



From the Hooke's law,

$$T_3 = \frac{4\lambda(x - 3l)}{3l} \qquad\qquad T_4 = \frac{\lambda(8l - x - 2l)}{2l}$$

$$\Rightarrow T_4 = \frac{\lambda(6l - x)}{2l}$$

Applying $\rightarrow \underline{F} = m\underline{a}$ to the particle

$T_4 - T_3 = m\ddot{x}$

$\Rightarrow \dfrac{\lambda(6l - x)}{2l} - \dfrac{4\lambda(x - 3l)}{3l} = m\ddot{x}$

$\Rightarrow \dfrac{\lambda}{6l}(18l - 3x - 8x + 24l) = m\ddot{x}$

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$\Rightarrow m\ddot{x} = \frac{\lambda}{6l}(42l - 11x)$

$\Rightarrow m\ddot{x} + \frac{11\lambda}{6l}\left(x - \frac{42}{11}l\right) = 0$

$\Rightarrow \ddot{x} + \frac{11\lambda}{6ml}\left(x - \frac{42}{11}l\right) = 0$

Let $y = x - \frac{42}{11}l \Rightarrow \ddot{y} = \ddot{x}$

$\therefore \ddot{y} + \frac{11\lambda}{6ml}y = 0$

This is of the form $\ddot{y} + \omega^2 y = 0$. Therefore the particle executes simple harmonic motion with $\omega = \sqrt{\frac{11\lambda}{6ml}}$

and the centre given by $y = 0 \Rightarrow x = \frac{42}{11}l$

Let O be the centre of the above SHM, $AO = \frac{42}{11}l$.

Since the particle is released from M, where $AM = 4l$, at $x = 4l$ the velocity of the particle is zero.

If p is the amplitude of this motion, $p = AM - AO \Rightarrow p = 4l - \frac{42}{11}l \Rightarrow p = \frac{2}{11}l$.

Velocity v of the particle is given by $v^2 = \omega^2\left(p^2 - y^2\right)$, where $p = \frac{2}{11}l$.

When $x = \frac{41}{11}l$, $y = \frac{41}{11}l - \frac{42}{11}l = -\frac{l}{11}$

Velocity of the particle P when it is at a point, distance $\frac{41}{11}l$ from $A = \omega\sqrt{\left(\left(\frac{2l}{11}\right)^2 - \left(\frac{-l}{11}\right)^2\right)}$

$$= \sqrt{\frac{11\lambda}{6ml}}\sqrt{\frac{3l^2}{121}}$$

$$= \sqrt{\frac{11\lambda \times 3l^2}{6ml \times 121}}$$

$$= \sqrt{\frac{\lambda l}{22m}}$$

Let us assume that the solutions of the equation $\ddot{y} + \frac{11\lambda}{6ml}y = 0$ is of the form $y = A\cos\omega t + B\sin\omega t$, where A and B are constants and $\omega = \sqrt{\frac{11\lambda}{6ml}}$.

$y = A\cos\omega t + B\sin\omega t$

Differentiating with respect to t

$\dot{y} = -A\omega\sin\omega t + B\omega\cos\omega t$

Since the particle is released from rest, when $t = 0$, $\dot{y} = 0$.

$\Rightarrow 0 = -A\omega\sin(0) + B\omega\cos(0)$

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$\Rightarrow B\omega = 0$

$\Rightarrow B = \underline{\underline{0}} \; (\because \omega \neq 0)$

$\therefore y = A\cos\omega t$

When $t = 0$, $x = 4l$. *i.e.* $y = 4l - \frac{42}{11}l = \frac{2}{11}l$

$\Rightarrow \frac{2}{11}l = A\cos(0)$

$\Rightarrow A = \underline{\underline{\frac{2l}{11}}}$

$\omega = \underline{\underline{\sqrt{\frac{11\lambda}{6ml}}}}$

Therefore the values of the constants are $A = \frac{2l}{11}$, $B = 0$ and $\omega = \sqrt{\frac{11\lambda}{6ml}}$.

14.

(a) Let *A* and *B* be two distinct points not collinear with a point *O*. Let the position vectors of the points *A* and *B* with respect to the point *O* be **a** and **b** respectively. If *D* is the point on *AB* such that $BD = 2DA$, show that the position vector of the point *D* with respect to the point *O* is $\frac{1}{3}(2\mathbf{a} + \mathbf{b})$.

If $\overrightarrow{BC} = k\mathbf{a}$, $(k > 1)$ and the point *O, D* and *C* are collinear, find the value of *k* and the ratio $OD : DC$.

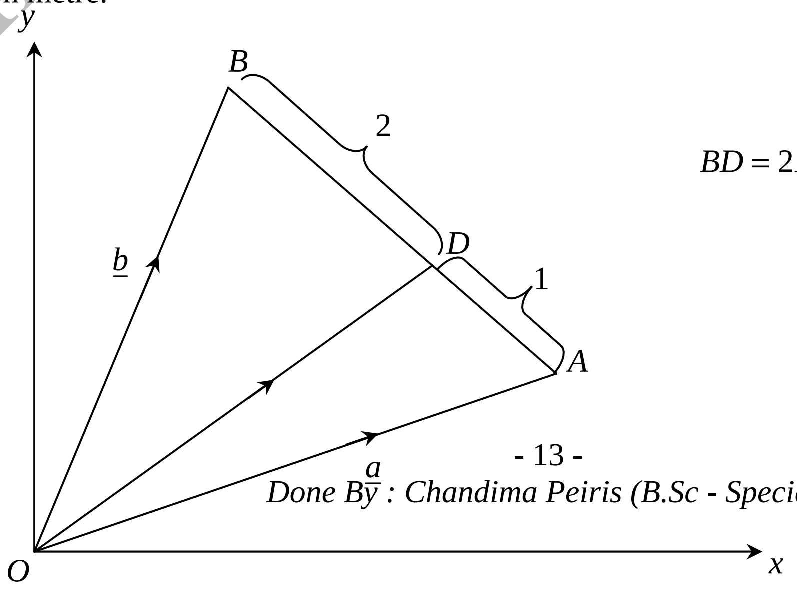
Express $\overrightarrow{AC}$ in terms of **a** and **b**.
Further, if the line through the point *O* parallel to *AC* meets *AB* at *E*, show that $6DE = AB$.

(b) The coordinates of the points *A*, *B* and *C* with respect to a rectangular Cartesian axes *Ox* and *Oy*, are $(\sqrt{3}, 0)$, $(0, -1)$ and $\left(\frac{2\sqrt{3}}{3}, 1\right)$ respectively. Forces of magnitude 6*P*, 4*P*, 2*P* and $2\sqrt{3}P$ newtons act along *OA*, *BC*, *CA* and *BO* respectively in the directions indicated by the order of the letters. Find the magnitude and the direction of the resultant of these forces.
Find the position at which the line of action of the resultant cuts the *y*-axis.
Hence, find the equation of the line of action of the resultant.

Another force of magnitude $6\sqrt{3}P$ newtons is introduced to the system along *AB* in the direction indicated by the order of the letters. Show that the system is reduced to a couple of magnitude 10*P* newton metre.

Answer

(a)



$BD = 2DA \Rightarrow BD : DA = 2 : 1$

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$\vec{OA} = \underline{a}$, $\vec{OB} = \underline{b}$

$\vec{AB} = \vec{AO} + \vec{OB}$

$= \underline{b} - \underline{a}$

Since $AD = \frac{1}{3}AB$ and *A*, *D* and *B* are collinear *i.e.*

$\vec{AD} = \frac{1}{3}\vec{AB}$

$\vec{AD} = \frac{1}{3}(\underline{b} - \underline{a})$

$\vec{OD} = \vec{OA} + \vec{AD}$

$= \underline{a} + \frac{1}{3}(\underline{b} - \underline{a})$

$= \frac{3\underline{a} + \underline{b} - \underline{a}}{3}$

$\therefore \vec{OD} = \frac{1}{3}(2\underline{a} + \underline{b})$

$\therefore$ The position vector of the point *D* with respect to the point $O = \frac{1}{3}(2\underline{a} + \underline{b})$

Let $\vec{BC} = k\underline{a}$ $(k > 1)$

$\vec{OC} = \vec{OB} + \vec{BC} \Rightarrow \vec{OC} = k\underline{a} + \underline{b}$

$\vec{OD} = \frac{1}{3}(2\underline{a} + \underline{b})$

Since *O*, *D* and *C* are collinear, $\vec{OC} = \lambda \vec{OD}$, where $\lambda \in \Re$.

$\Rightarrow k\underline{a} + \underline{b} = \frac{\lambda}{3}(2\underline{a} + \underline{b})$

$\Rightarrow \left(k - \frac{2\lambda}{3}\right)\underline{a} + \left(1 - \frac{\lambda}{3}\right)\underline{b} = \underline{0}$

$\Rightarrow k - \frac{2\lambda}{3} = 0$ and $1 - \frac{\lambda}{3} = 0$

$\Rightarrow k = \frac{2\lambda}{3}$ and $\lambda = 3$

$\Rightarrow k = 2$ and $\lambda = 3$

$\therefore k = \underline{\underline{2}}$

Since $k = 2$, $\vec{OC} = 2\underline{a} + \underline{b}$.

$\vec{DC} = \vec{DO} + \vec{OC}$

$= -\frac{1}{3}(2\underline{a} + \underline{b}) + 2\underline{a} + \underline{b}$

$= \frac{-2\underline{a} - \underline{b} + 6\underline{a} + 3\underline{b}}{3}$

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$= \dfrac{4\underline{a} + 2\underline{b}}{3}$

$= \dfrac{2}{3}(2\underline{a} + \underline{b})$

$\Rightarrow \overrightarrow{DC} = 2\overrightarrow{OD}$

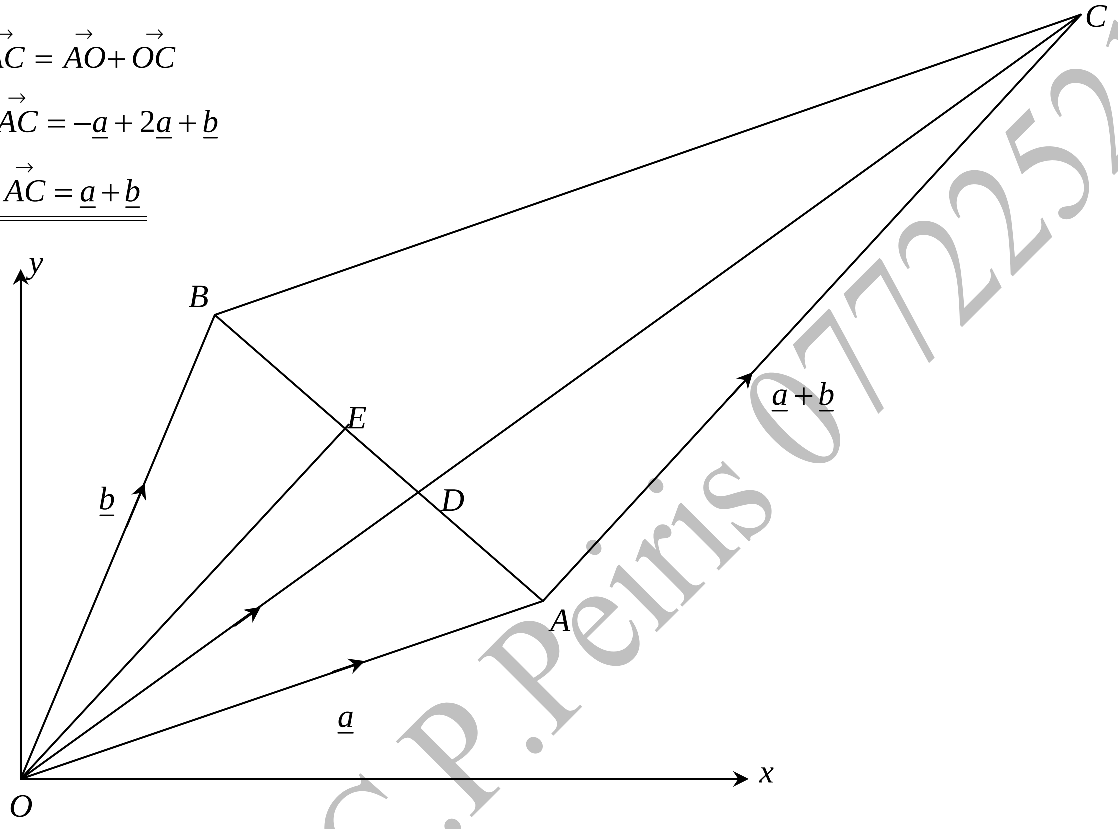
$\therefore DC = 2OD$

$\Rightarrow \underline{\underline{OD : DC = 1:2}}$

$\therefore \overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC}$

$\Rightarrow \overrightarrow{AC} = -\underline{a} + 2\underline{a} + \underline{b}$

$\Rightarrow \underline{\underline{\overrightarrow{AC} = \underline{a} + \underline{b}}}$



Since $AC \nearrow\!\!\nearrow OE$, $\overrightarrow{OE} = \mu \overrightarrow{AC}$ where μ is a constant.

$\therefore \overrightarrow{OE} = \mu(\underline{a} + \underline{b})$

$\overrightarrow{AE} = \overrightarrow{AO} + \overrightarrow{OE}$

$\Rightarrow \overrightarrow{AE} = -\underline{a} + \mu\underline{a} + \mu\underline{b}$

$\overrightarrow{AD} = \overrightarrow{AO} + \overrightarrow{OD}$

$\Rightarrow \overrightarrow{AD} = -\underline{a} + \dfrac{1}{3}(2\underline{a} + \underline{b}) = \dfrac{\underline{b} - \underline{a}}{3}$

Since *A*, *D* and *E* are collinear $\overrightarrow{AD} = \gamma \overrightarrow{AE}$, where $\gamma \in \Re$.

$\dfrac{\underline{b} - \underline{a}}{3} = \gamma(-\underline{a} + \mu\underline{a} + \mu\underline{b})$

$\Rightarrow \left(\dfrac{1}{3} - \gamma\mu\right)\underline{b} + \left(\gamma - \gamma\mu - \dfrac{1}{3}\right)\underline{a} = \underline{0}$

$\Rightarrow \gamma\mu - \dfrac{1}{3} = 0$ and $\gamma - \gamma\mu - \dfrac{1}{3} = 0$

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$\Rightarrow \gamma\mu = \frac{1}{3}$ and $\gamma - \frac{1}{3} - \frac{1}{3} = 0$

$\Rightarrow \gamma\mu = \frac{1}{3}$ and $\gamma = \frac{2}{3}$

$\Rightarrow \frac{2\mu}{3} = \frac{1}{3}$ and $\gamma = \frac{2}{3}$

$\Rightarrow \mu = \frac{1}{2}$ and $\gamma = \frac{2}{3}$

$\therefore \vec{OE} = \frac{1}{2}(\underline{a} + \underline{b})$

$\vec{DE} = \vec{DO} + \vec{OE} = -\frac{1}{3}(2\underline{a} + \underline{b}) + \frac{1}{2}(\underline{a} + \underline{b})$

$\Rightarrow \vec{DE} = \frac{1}{6}(-4\underline{a} - 2\underline{b} + 3\underline{a} + 3\underline{b})$

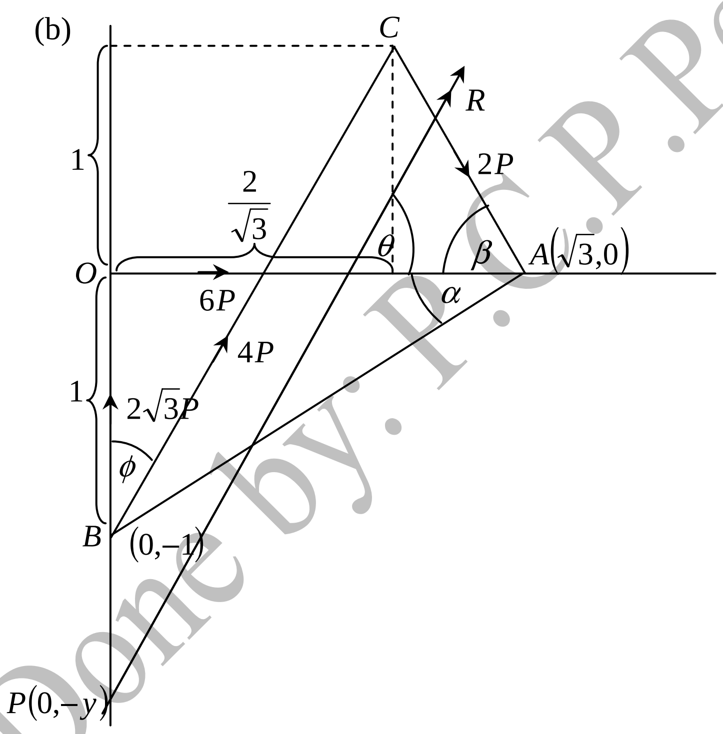
$\Rightarrow \vec{DE} = \frac{1}{6}(\underline{b} - \underline{a})$

$\Rightarrow \vec{DE} = \frac{1}{6}\vec{AB}$

$\therefore DE = \frac{1}{6}AB$ ($\because$ *A*, *B*, *E* and *D* are collinear)

$\Rightarrow \underline{\underline{AB = 6DE}}$

(b)



Let *R* be the resultant of the system of forces and it makes an angle θ with the horizontal.

$\tan\phi = \frac{2}{\sqrt{3}\times 2} = \frac{1}{\sqrt{3}}$

$\Rightarrow \phi = 30^\circ$

$\tan\alpha = \frac{1}{\sqrt{3}}$

$\Rightarrow \alpha = 30^\circ$

$\tan\beta = \frac{1}{\sqrt{3} - \frac{2}{\sqrt{3}}} = \frac{\sqrt{3}}{3-2} = \sqrt{3}$

$\Rightarrow \beta = 60^\circ$

Resolving $\uparrow$ $R\sin\theta = 2\sqrt{3}P + 4P\cos\phi - 2P\sin\beta$

$= 2\sqrt{3}P + 4P\times\frac{\sqrt{3}}{2} - 2P\times\frac{\sqrt{3}}{2}$

$R\sin\theta = 3\sqrt{3}P$ ------(1)

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Resolving $\rightarrow R\cos\theta = 6P + 2P\cos\beta + 4P\sin\phi$

$$= 6P + 2P\times\frac{1}{2} + 4P\times\frac{1}{2}$$

$$R\cos\theta = 9P ------(2)$$

$(1)^2 + (2)^2;\ R^2 = \left(3^2\times 3 + 3^2\times 3^2\right)P^2 \Rightarrow R^2 = 3^2\times 2^2\times P^2\times 3$

$$\therefore R = 6\sqrt{3}P$$

$\frac{(1)}{(2)};\ \tan\theta = \frac{3\sqrt{3}}{9} = \frac{1}{\sqrt{3}}$

$\therefore \theta = 30^\circ$

Magnitude of the resultant $= 6\sqrt{3}P$

Direction of the resultant $= 30^\circ$ with the horizontal

Let $P(0,-y)$ be the point of intersection of the line of action of the resultant and the y-axis.
Taking moments about O

$\circlearrowleft O\ R\sin(90^\circ-\theta)\times y = 4P\sin\phi\times 1 - 2P\cos\beta\times 1 - 2P\sin\beta\times\frac{2}{\sqrt{3}}$

$\Rightarrow R\cos\theta\times y = 4P\times\frac{1}{2}\times 1 - 2P\times\frac{1}{2}\times 1 - 2P\times\frac{\sqrt{3}}{2}\times\frac{2}{\sqrt{3}}$

$\Rightarrow 9Py = 2P - P - 2P$

$\Rightarrow 9y = -1$

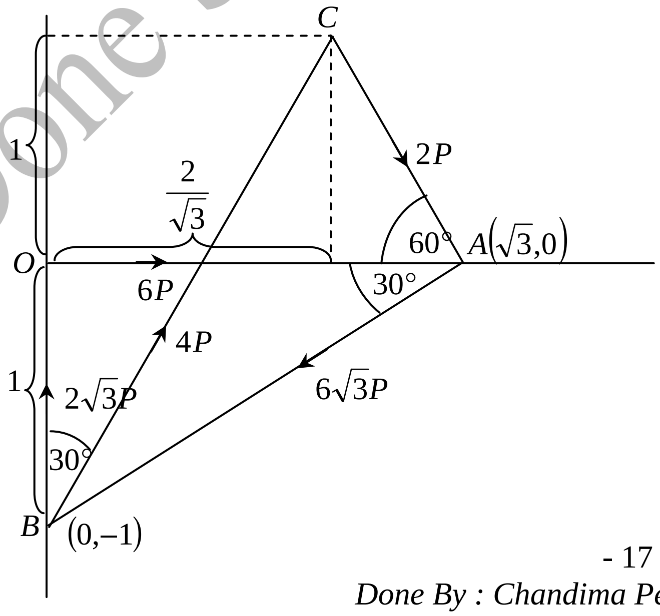
$\Rightarrow y = -\frac{1}{9}$

$\therefore$ The point at which the line of action of the resultant cuts the y-axis $= \left(0,-\frac{1}{9}\right)$

Let $y = mx + c$ be the equation of the line of action of the resultant.

Here $m = \tan\theta = \tan 30^\circ = \frac{1}{\sqrt{3}}$ and $c = -\frac{1}{9}$.

$\therefore$ The equation of the line of action of the resultant $\equiv y = \frac{1}{\sqrt{3}}x - \frac{1}{9}$



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$\overrightarrow{X} = 6P + 4P\times\frac{1}{2} + 2P\times\frac{1}{2} - 6\sqrt{3}P\times\frac{\sqrt{3}}{2} = 6P + 2P + P - 9P$

$\Rightarrow \overrightarrow{X} = 0$

$\uparrow Y = 2\sqrt{3}P + 4P\times\frac{\sqrt{3}}{2} - 2P\times\frac{\sqrt{3}}{2} - 6\sqrt{3}P\times\frac{1}{2} = 2\sqrt{3}P + 2\sqrt{3}P - \sqrt{3}P - 3\sqrt{3}P$

$\Rightarrow \uparrow Y = 0$

$$\overset{\curvearrowleft}{G_o} = 4P\times\frac{1}{2}\times 1 - 2P\times\frac{1}{2}\times 1 - 2P\times\frac{\sqrt{3}}{2}\times\frac{2}{\sqrt{3}} - 6\sqrt{3}P\times\frac{1}{2}\times\sqrt{3}$$
$$= 2P - P - 2P - 9P$$

$$\overset{\curvearrowleft}{G_o} = -10P$$

Here $\overrightarrow{X} = 0$, $\uparrow Y = 0$ and $\overset{\curvearrowleft}{G_o} \neq 0$, the system of forces is reduced to a couple.

Magnitude of the couple $= \underline{\underline{10P}}$

Direction of the couple = <u>Clockwise</u>

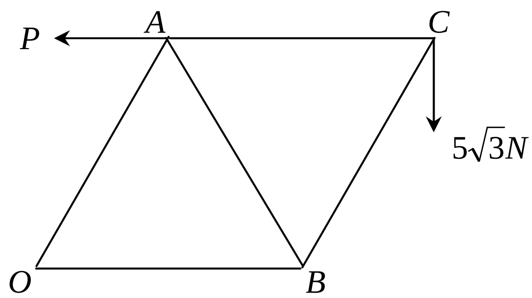
15.

(a) Two equal uniform rods AB and AC each of weight W are freely jointed at A and the ends B and C are connected by a light inextensible string. The rods are kept in equilibrium in a vertical plane with the ends B and C are on two smooth planes each of which inclined at an angle α to the horizontal; BC being horizontal and A being above BC. Find the reaction at B.

If $\tan\theta > 2\tan\alpha$, where $B\hat{A}C = 2\theta$, then show that the tension of the string is $\frac{1}{2}W(\tan\theta - 2\tan\alpha)$.

Find the reaction at the joint A.

(b) Five light equal rods OA, OB, AC, AB and BC are smoothly jointed at their ends to form a framework as shown in the figure.



The framework is smoothly hinged at O and carries a weight $5\sqrt{3}$ newtons at C. The framework is held in a vertical plane, with OB horizontal by a horizontal force P newtons at A.

(i) Find the value of P.

(ii) Find the magnitude and the direction of the reaction at O.

(iii) Using Bow's notation, draw a stress diagram for the framework and find the stresses in all rods, distinguishing between tensions and thrusts.

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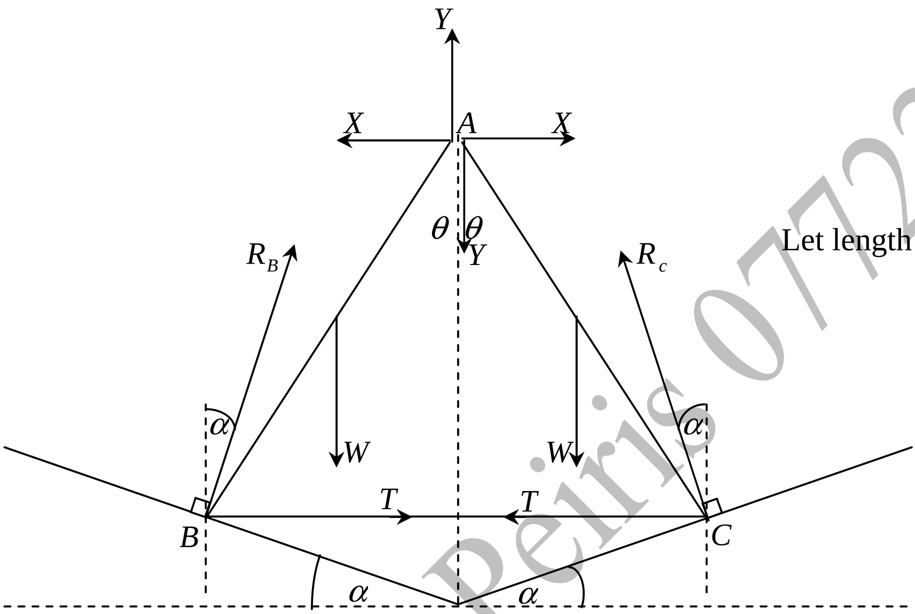
Answer

(a)



Let length of the rod AB is $2a$.

Taking moments about B for the rod BA

B ↺ $X\times 2a\cos\theta + Y\times 2a\sin\theta - W\times a\sin\theta = 0 ------(1)$

Taking moments about C for the rod AC

C ↺ $-X\times 2a\cos\theta + Y\times 2a\sin\theta + W\times a\sin\theta = 0 ------(2)$

$(1)+(2);\ 4Ya\sin\theta = 0 \Rightarrow Y = 0$

$(1)-(2);\ 4Xa\cos\theta = 2Wa\sin\theta$

$\Rightarrow X = \frac{W}{2}\tan\theta$

Resolving ↑ for the rod AB

$R_B\cos\alpha = W$

$\Rightarrow R_b = W\sec\alpha$

$\therefore$ Reaction at the point $B = \underline{\underline{W\sec\alpha}}$

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Resolving $\rightarrow$ for the rod AB

$T + R_B \sin\alpha = X$

$\Rightarrow T + \dfrac{W}{\cos\alpha} = \dfrac{W}{2}\tan\theta$

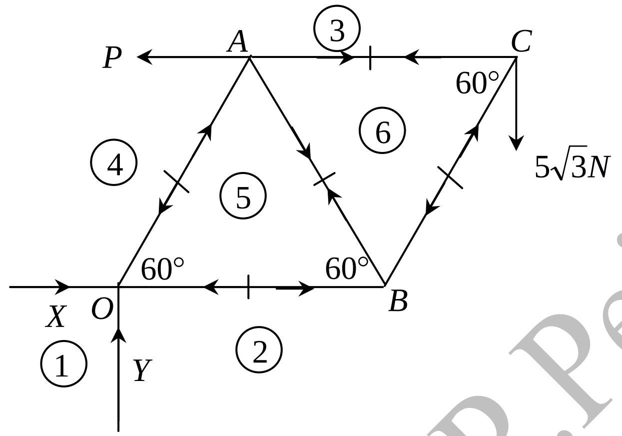
$\Rightarrow T = \dfrac{W}{2}\left(\tan\theta - 2\tan\alpha\right)$

$\therefore$ The tension in the string $= \dfrac{W}{2}\left(\tan\theta - 2\tan\alpha\right)$

Magnitude of the reaction at the joint $A = \dfrac{W}{2}\tan\theta$

Direction of the reaction at the joint A = Horizontal direction

(b)



Since the frameworks is in equilibrium,

Resolving $\rightarrow$, $X = P$ ----(1) Resolving $\uparrow$, $Y = 5\sqrt{3}N$ ----(2)

Taking moments about O

$\overset{\curvearrowleft}{O}$ $P \times 2a\sin 60^\circ = 5\sqrt{3} \times 3a$, where $2a$ is the length of the rod OA.

$\Rightarrow P \times 2a \times \dfrac{\sqrt{3}}{2} = 5\sqrt{3} \times 3a$

$\Rightarrow P = 15N$

(i) The value of $P = 15N$

Form (1); $X = 15N$

(ii) Magnitude of the reaction at $O = \sqrt{X^2 + Y^2}$

$= \sqrt{15^2 + 5^2 \times 3}$

$= \sqrt{5^2 \times 2^2 \times 3}$

$= 10\sqrt{3}N$

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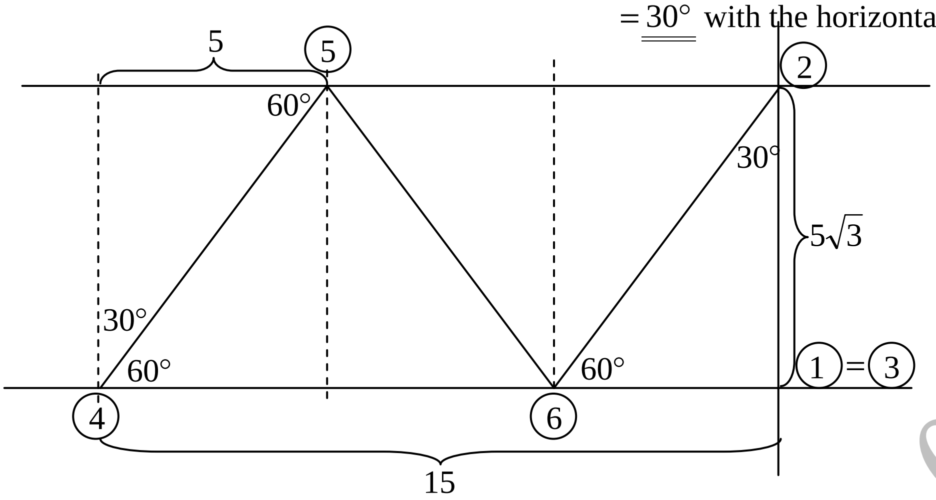
Direction of the reaction at O $= \tan^{-1}\left(\frac{Y}{X}\right)$

$= \tan^{-1}\left(\frac{5\sqrt{3}}{15}\right)$

$= \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$= \underline{\underline{30^\circ}}$ with the horizontal



$(4)-(5)=(2)-(6)=\sqrt{5^2\times 3+5^2}=10N$

$(5)-(6)=10N$

$(3)-(6)=5N$

$(2)-(5)=10N$

Rod	Magnitude	Nature
OA	$10N$	Thrust
OB	$10N$	Thrust
AC	$5N$	Tension
AB	$10N$	Tension
BC	$10N$	Thrust

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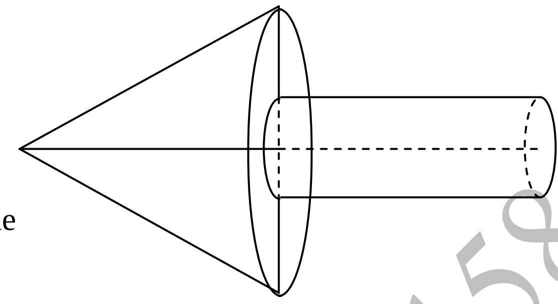
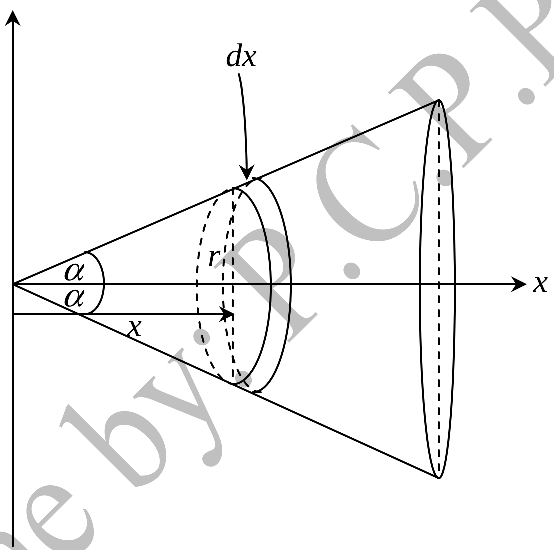
16. Show that the centre of mass of a uniform solid right circular cone of height h is on its axis of symmetry at a distance $\frac{1}{4}h$ from the base of the cone.



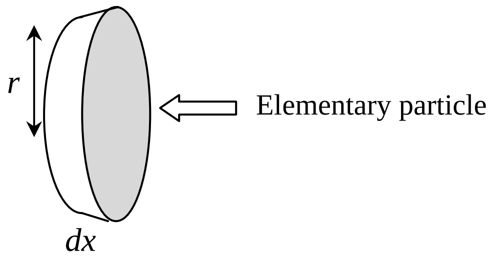
A uniform solid composite body consists of a right circular cone of base radius $3r$ and height h and a right circular cylinder of radius r and height $2h$ fixed together as shown in the figure.

Show that the centre of mass of the composite body is on its axis of symmetry at a distance $\frac{5}{4}h$ from the vertex of the cone.
The composite body is hanged freely in a vertical plane by a light inextensible string, one end of which is fixed to a ceiling and the other end to a point A on the circumference of the circular base of the cone.
If the axis of symmetry of the composite body makes an angle α with the downward vertical, show that, $\tan\alpha = \frac{12r}{h}$.
By applying along the axis of symmetry of the composite body, a force P at the vertex of the cone, the composite body is kept in equilibrium so that the axis of symmetry of the composite body is horizontal. Find the force P and the tension of the string in terms of W and α, where W is the weight of the composite body.

Answer



$r = \tan\alpha$



Since the cone is symmetrical about the x-axis, its centre of gravity should lies on the x-axis. Then let $G \equiv (\bar{x}, 0)$. Consider the elementary particle in the shape of the cylinder with height dx and radius r, at a distance x from the vertex.
Let dm ne its mass and ρ be its density.

$$dm = \pi r^2 dx\rho \Rightarrow dm = \pi x^2 \tan^2 \alpha dx$$

When x varies from 0 to h the whole object can be obtained.
By the definition of centre of gravity,

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$$\bar{x} = \frac{\int_0^h x\,dm}{\int_0^h dm}$$

$$\Rightarrow \bar{x} = \frac{\int_0^h \pi x^3 \tan^2\alpha \rho\, dx}{\int_0^h x^2 \tan^2\alpha \rho\, dx} = \frac{\pi \tan^2\alpha \rho \int_0^h x^3 dx}{\pi \tan^2\alpha \rho \int_0^h x^2 dx}$$

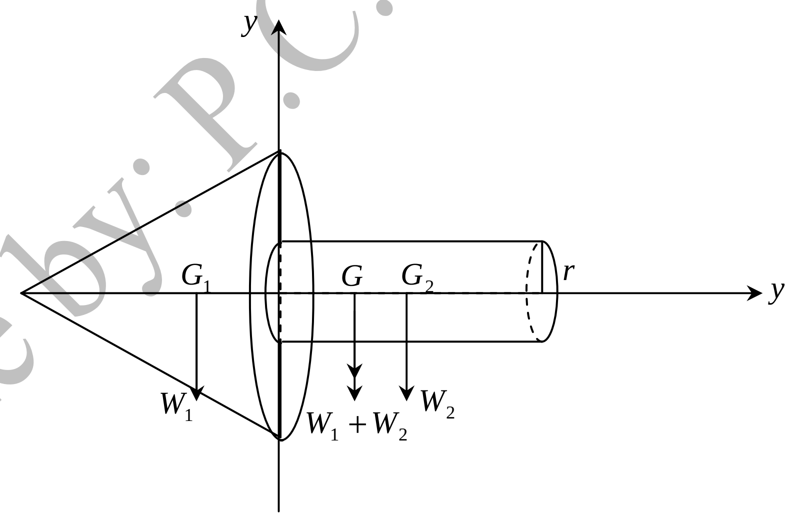
$$\Rightarrow \bar{x} = \frac{\int_0^h x^3 dx}{\int_0^h x^2 dx}$$

$$\Rightarrow \bar{x} = \frac{\left[\frac{x^4}{4}\right]_0^h}{\left[\frac{x^3}{3}\right]_0^h}$$

$$\Rightarrow \bar{x} = \frac{h^4}{4} \times \frac{3}{h^3}$$

$$\therefore \bar{x} = \frac{3h}{4}$$

$\therefore$ The centre of gravity of a uniform solid right circular cone of height h is on its axis of symmetry at a distance $\frac{1}{4}h$ from the base of the cone.



This composite body is symmetrical about x-axis. $\therefore$ The centre of gravity of it should lies on the x-axis.
Let ρ be the density of the composite body.

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Body	Weight	Coordinates of the centre of gravity
Cone	$\frac{1}{3}\pi \times 9r^2 \times h\rho g$ $W_1 = 3\pi r^2 h\rho g$	$G_1 \equiv \left(-\frac{h}{4}, 0\right)$
Cylinder	$\pi r^2 \times 2h\rho g$ $W_2 = 2\pi r^2 h\rho g$	$G_2 \equiv (h, 0)$
Composite body	$W_1 + W_2 = 5\pi r^2 h\rho g$	$G \equiv (\bar{x}, 0)$

$$5\pi r^2 h\rho g \times \bar{x} = 3\pi r^2 h\rho g \times \left(\frac{-h}{4}\right) + 2\pi r^2 h\rho g \times h$$

$$\Rightarrow 5\bar{x} = -\frac{3h}{4} + 2h = \frac{5h}{4}$$

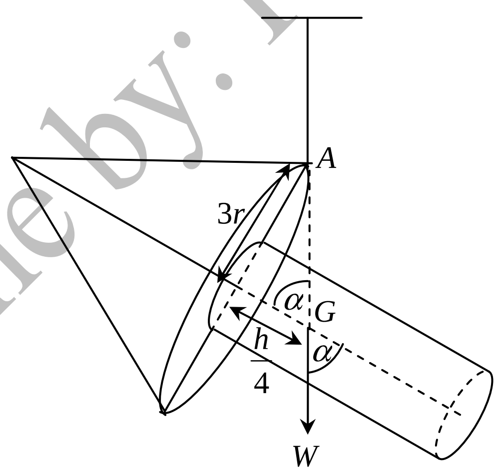
$$\therefore \bar{x} = \frac{h}{4}$$

$\therefore$ Distance to the centre of gravity of the composite body from the vertex of the cone $= h + \frac{h}{4}$

$$= \frac{5h}{4}$$

$\therefore$ The centre of mass of the composite body is on its axis of symmetry at a distance $\frac{5}{4}h$ from the vertex of the cone.

When the composite body is hang freely in a vertical plane by a light inextensible string, one end of which is fixed to a ceiling and the other end to a point A on the circumference of the circular base of the cone.
Let α is the angle made by the axis of symmetry of the composite body with the downward vertical.



$$\tan\alpha = \frac{3r}{\frac{h}{4}}$$

$$\tan\alpha = \frac{12r}{h}$$

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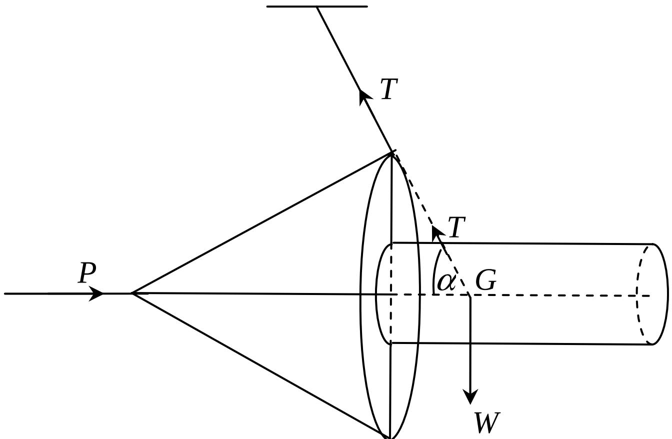

When the composite body is in equilibrium, the point G is also in equilibrium under the action of three forces T, W and P, where T is the tension in the string.

Applying Lami's theorem to the point G

$$\frac{P}{\sin(90^\circ + \alpha)} = \frac{T}{\sin 90^\circ} = \frac{W}{\sin \alpha}$$

$$\Rightarrow \frac{P}{\cos \alpha} = \frac{T}{1} = \frac{W}{\sin \alpha}$$

$$\Rightarrow P = W \cot \alpha \text{ and } T = W \operatorname{cosec} \alpha$$

The value of the force P = $\underline{\underline{W \cot \alpha}}$

Tension in the string = $\underline{\underline{W \operatorname{cosec} \alpha}}$

17.

(a) An urn contains 5 white, 3 black and 7 red similar balls. Three balls are taken from the urn at random without replacement.
Find the probability that
- (i) all three balls are black,
- (ii) none of the three balls is white,
- (iii) at least one ball is white,
- (iv) the balls are of different colours,
- (v) the three balls are taken in the order black, red then white.

(b) Students in a certain class were given a question paper in Statistics. The marks obtained by these students are given in the following grouped frequency table.

Range of Marks	Number of students
00-20	14
20-40	f_1
40-60	27
60-80	f_2
80-100	15

The frequencies of the marks ranges 20-40 and 60-80 are missing in the table. However, the mode and the median of the grouped frequency distribution are known as 48 and 50 respectively.
Calculate the two missing frequencies in the table.
Hence, obtain the total number of students who sat for the Statistics paper.
Find the mean and the standard deviation of the grouped frequency distribution.

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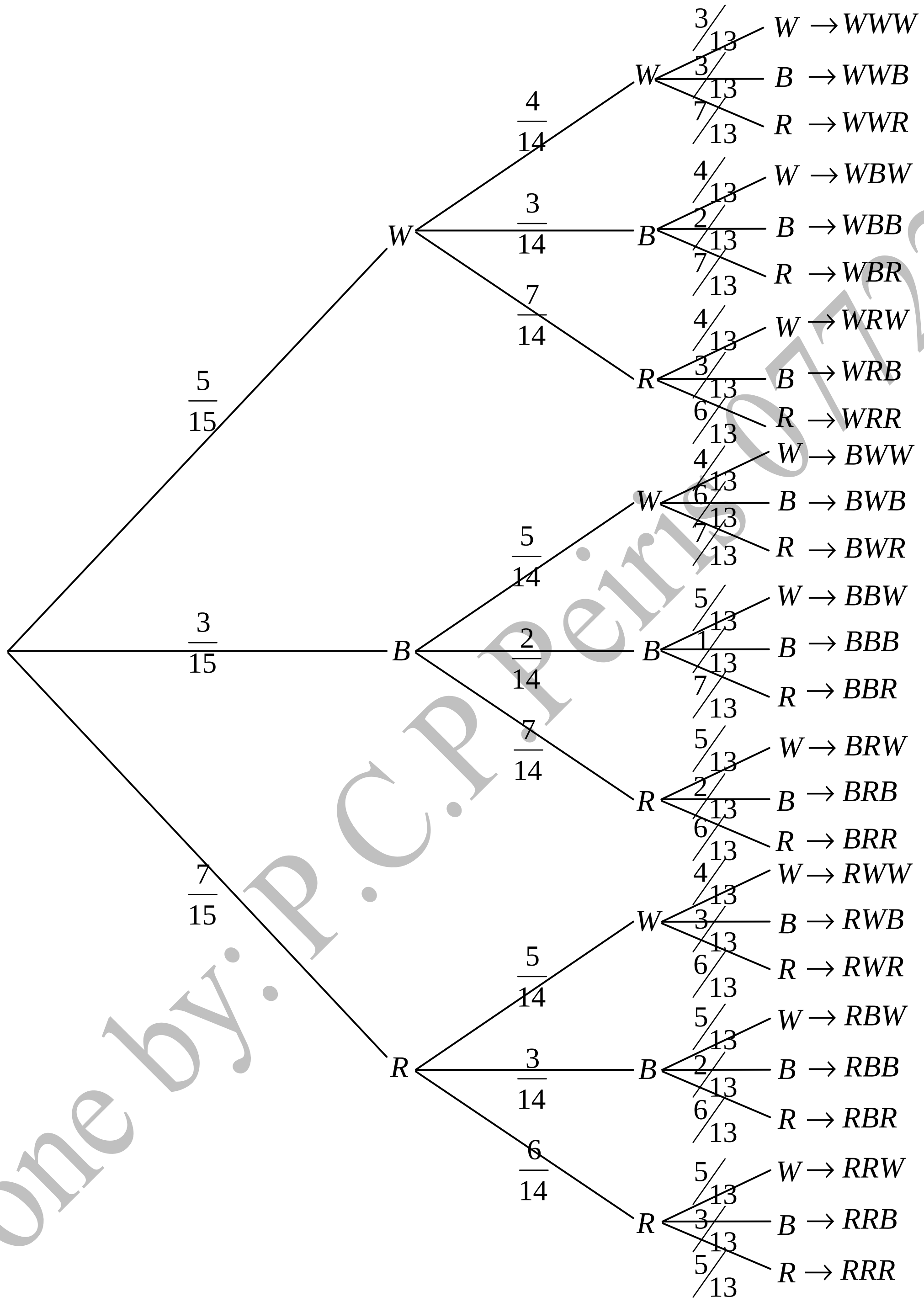
Answer

(a)

$W - 5$
$B - 3$
$R - 7$

Let W - The event that getting white ball.
B - The event that getting black ball.
R - The event that getting red ball.



(i) Probability that all three balls are black $= P(BBB)$

$$= \frac{3}{15} \times \frac{2}{14} \times \frac{1}{13}$$

$$= \frac{1}{455}$$

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(ii) Probability that of none of the three balls is white $= P(BBB) + P(BBR) + P(BRB) + P(BRR) + P(RBB) + P(RBR) + P(RRB) + P(RRR)$

$$= \frac{1}{455} + \frac{3\times2\times7}{15\times14\times13} + \frac{3\times7\times2}{15\times14\times13} + \frac{3\times7\times6}{15\times14\times13}$$
$$+ \frac{7\times3\times2}{15\times14\times13} + \frac{7\times3\times6}{15\times14\times13} + \frac{7\times6\times3}{15\times14\times13} + \frac{7\times6\times5}{15\times14\times13}$$
$$= \frac{1+7+7+21+7+21+21+35}{455}$$
$$= \frac{120}{455}$$
$$= \underline{\underline{\frac{24}{91}}}$$

(iii) Probabilty that at least one ball is white = 1 - Probability that none of the three balls is white

$$= 1 - \frac{24}{91}$$
$$= \underline{\underline{\frac{67}{91}}}$$

(iv) Probability that the balls are of different colours $= P(WRB) + P(WBR) + P(BWR) + P(BRW) + P(RBW) + P(RWB)$

$$= \frac{5\times7\times3}{15\times14\times13} + \frac{5\times3\times7}{15\times14\times13} + \frac{3\times5\times7}{15\times14\times13}$$
$$+ \frac{3\times7\times5}{15\times14\times13} + \frac{7\times3\times5}{15\times14\times13} + \frac{7\times5\times3}{15\times14\times13}$$
$$= \frac{15\times7}{15\times14\times13}\times6$$
$$= \frac{6}{2\times13}$$
$$= \underline{\underline{\frac{3}{13}}}$$

(v) Probability that the three balls are taken in the order black, red and white $= P(BRW)$

$$= \frac{3}{15}\times\frac{7}{14}\times\frac{5}{13} = \underline{\underline{\frac{1}{26}}}$$

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(b)

Range of Marks	Number of students	Cumulative frequency
00 - 20	14	14
20 - 40	f_1	$14+f_1$
40 - 60	27	$41+f_1$
60 - 80	f_2	$41+f_1+f_2$
80 - 100	15	$56+f_1+f_2$

Since the mode is 48, the model class is 40 - 60.

Mode $=L+C\left(\dfrac{\Delta_1}{\Delta_1+\Delta_2}\right)$, in the usual meaning.

Mode = 48, $L=40$, $C=20$, $\Delta_1=27-f_1$, $\Delta_2=27-f_2$

$$48=40+20\left(\frac{27-f_1}{27-f_1+27-f_2}\right)$$

$$\Rightarrow 8=20\left(\frac{27-f_1}{54-f_1-f_2}\right)$$

$$\Rightarrow 2(54-f_1-f_2)=5(27-f_1)$$

$$\Rightarrow 108-2f_1-2f_2=135-5f_1$$

$$\Rightarrow 3f_1-2f_2=27------(1)$$

Since the median is 50, the class which contains the median is 40 - 60.

Median $=L+\dfrac{C_m}{f_m}\left(\dfrac{N}{2}-F_{m-1}\right)$, in the usual meaning.

Median = 50 , $L=40$, $C_m=20$, $f_m=27$, $N=56+f_1+f_2$, $F_{m-1}=14+f_1$

$$50=40+\frac{20}{27}\left(\frac{56+f_1+f_2}{2}-(14+f_1)\right)$$

$$\Rightarrow 10=\frac{20}{27}\left(\frac{56+f_1+f_2-28-2f_1}{2}\right)$$

$$\Rightarrow 27=28+f_2-f_1$$

$$\Rightarrow f_1-f_2=1------(2)$$

$(1)-2\times(2)$; $3f_1-2f_1=25$

$$\Rightarrow f_1=25 \text{ and } f_2=24$$

The value of $f_1=\underline{\underline{25}}$

The value of $f_2=\underline{\underline{24}}$

Total number of students who sat for the Statistics paper $=56+25+24$

$=\underline{\underline{105}}$

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Interval	x_i - Mid value	f_i	$u_i = \frac{x_i - 50}{20}$	u_i^2	$f_i u_i^2$	$f_i u_i$
00 - 20	10	14	-2	4	56	-28
20 - 40	30	25	-1	1	25	-25
40 - 60	50	27	0	0	0	0
60 - 80	70	24	1	1	24	24
80 - 100	90	15	2	4	60	30

$\sum_{i=1}^{5} f_i = 105$, $\sum_{i=1}^{5} f_i u_i = 1$, $\sum_{i=1}^{5} f_i u_i^2 = 165$

$\bar{x} = A + C\bar{u}$, where $A = 50$, $C = 20$ and $\bar{u} = \dfrac{\sum_{i=1}^{5} f_i u_i}{\sum_{i=1}^{5} f_i}$

$$\Rightarrow \bar{x} = 50 + 20\left(\frac{1}{105}\right)$$

$$\Rightarrow \bar{x} = 50.19$$

$\therefore$ Mean of the grouped frequency distribution $= \underline{\underline{50.19}}$

$$\sigma_x^2 = C^2 \sigma_u^2 = 20^2 \left[\frac{\sum_{i=1}^{5} f_i u_i^2}{\sum_{i=1}^{5} f_i} - \bar{u}^2 \right]$$

$$\Rightarrow \sigma_x^2 = 20^2 \left[\frac{165}{105} - \left(\frac{1}{105}\right)^2 \right]$$

$$\Rightarrow \sigma_x^2 \approx 20^2 \left(\frac{165}{105}\right) \ (\because \left(\frac{1}{105}\right)^2 \approx 0)$$

$$\Rightarrow \sigma_x = 20\sqrt{1.57}$$

$$\Rightarrow \sigma_x = 25.06$$

$\therefore$ Standard deviation of the grouped frequency distribution $= \underline{\underline{25.06}}$

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For A/L Combined Maths (Group/Individual) Classes 2013/ 2014
English and Sinhala Mediums

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